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Strategic Behavior under Intertemporal Production Externalities*

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Résumé / Abstract

Nous modélisons le jeu de choix optimal d'un input dont l'usage diminue l'efficacité dans le futur. Nous démontrons qu'il y a des équilibres multiples que l'on peut comparer en utilisant le critère de supériorité à la Pareto. La perte d'efficacité est plus grave si les firmes adoptent des stratégies markoviennes au lieu des stratégies à boucle ouverte.

We model the non-cooperative choice of levels of inputs whose current usage results in the future decline in their effectiveness. We show that there are multiple equilibria that are Pareto rankable. Compared with the social optimum, lack of cooperation implies excessive use of input, leading to excessively rapid rates of decline in effectiveness. The harm is more pronounced when firms use Markov perfect strategies, as compared with open-loop strategies.

Mots Clés : Jeux dynamiques, externalités

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1 Introduction

A good deal of attention has been paid by economists to the problems that arise when individuals have access to a resource stock, whether it be renewable like fish, or non-renewable like mineral deposits. In either case, individual exploiters may not have an incentive to incorporate into their calculations the costs imposed on others by their current exploitation. The presence of negative externalities establishes a presumption that current exploitation will, in the presence of free entry, tend to be too high, and the stock of the common resources depleted too quickly.

By contrast, there is another set of problems with a similar intertemporal common property flavour, believed by many scientists to have potentially serious implications for the welfare of future generations, but to which economists appear to have paid very little attention. We refer here for the tendency for the productivity of certain inputs to decline over time as the result of past use. Two particular examples come to mind. First, the medical profession and medical researchers have expressed alarm over the declining effectiveness over time of many antibiotics, such as sulfonamides and penicillin, as a result of their earlier use as therapeutic and prophylactic drugs. Second, similar fears have been expressed concerning declines in effectiveness of many pesticides, such as DDT. In both cases, current usage sets in train a process whereby over time the target population, be it of bacteria, rats, malarial mosquitos, sandflies and whatever, acquires resistance to the administered drug or poison, with the consequence that future doses are less effective and, indeed, may ultimately prove to be totally ineffective. This phenomenon, though inconvenient, does not by itself suggest that the use of such substances should be banned. However, insofar as individuals fail to take account of the later costs imposed on others as a consequence of their own current applications of an antibiotic or pesticide, there will be negative externalities and the current application may, from the social point of view, be inefficiently high. More generally, it raises questions concerning the design of mechanisms that provide incentives for individuals to behave in ways compatible with efficient intertemporal resource allocation.

The economic problems arising from the declining effectiveness of pesticides over time have been discussed by Feder and Regev (1975) and also by Regev, Shalit and Gutierrez (1983). Their model incorporate interesting features, such as predator-prey biological interaction and the dual dynamics of population growth and the evolution of the pest's resistance. On the other hand, their analysis of the competitive solution is oversimplified by their assumption that firms are static maximizers. In their models, firms do not take any account whatsoever of the effects of their actions on the dynamics of the system. Our paper attempts to address this neglected problem. We develop two models (one set in discrete time, one in continuous time) of dynamic strategic interactions among n players (firms, or individuals) that maximize their intertemporal objective function by choosing levels of an input whose use causes a steady decline in the effectiveness of the input. We analyse the equilibria of these games. For each model, we compare the social optimum with the outcomes of the non-cooperative game, where the concepts of open-loop Nash equilibrium and Markov-perfect Nash equilibrium are used alternatively. These concepts will be explained when the models are introduced.

Formally, our models are similar, but not identical, to differential games models of the fishery or other common property resources. (See Clemhout and Wan (1994).) There are at least two differences. First, in our models, the current profit of the firm depends on a multiplicative interaction between the state variable (the level effectiveness of a drug, say) and the control variable (the number of doses). Thus the state variable acts as a quality index. Second, a distinguishing feature of the discrete time model we use in this paper, which makes it really different from discrete time model of resource exploitation, is that at any time t, given that the effectiveness of the drug is positive, there is no upper bound on the quantity x_{jt} that firm j can choose. (In the case of resource exploitation in discrete time, the sum of the quantities extracted at time t cannot exceed the current stock of the resource.) This feature makes the dynamics of our models more complicated, and gives rise to a multiplicity of equilibrium strategy profiles, with outcomes that are Pareto rankable.

The paper is organized as follows. In section 2, we present a brief history of antibiotics and pesticides, and highlight the importance of the problem of resistance. In section 3, we develop a discrete time model involving n firms, and 3 periods. A continous time model is analysed in section 4. Section 5 contains concluding remarks.

2 A Brief History of Antibiotics and Pesticides

Mankind's experience with penicillin and dichlorodiphenyl trichloroethylene (DDT) have interesting and significant parallels. Both were developed and became available in significant quantities during the 1940's, and the development of both substances was greeted with hyperbole and greatly exaggerated claims of their likely beneficial effects. During the early years of their use, both were held up as powerful symbols of our ability to apply our growing understanding of chemical and biological phenomena to defeat some of the major scourges in human history. Each is the best-known example of a wide range of substances whose usage over time have generated a similar range of problems - problems that have dampened earlier enthousiasm and, indeed, have provoked genuine alarm in some quarters.

2.1 The Pesticide Story

DDT was first synthesized as long ago as 1874 by a Viennese pharmacist, Othmar Ziedler.However, its discovery was first registered in 1940 by a Swiss chemist, Paul H. Müller, who was at that time employed to find a chemical method for controlling clothes moth. In 1945, it was used with great success in a large-scale trial to control malaria along the Tennessee River. Such was the promise of DDT that it gained for Müller a Nobel prize in 1948. It became a central weapon in the World Health Organization's "Global Eradication of Malaria Plan" and of India's National Malaria Eradication Program, to which the United States supplied over two hundred thousand tons of DDT a year. According to Desowitz (1991), at its peak the American annual production of DDT exceeded 400,000 tons. As a bonus, it proved to be equally effective, if not more so, in controlling the insect vectors carrying other human diseases, such as the sandflies responsible for transmitting kala-azar, a significant killer in the Indian subcontinent.

The acquired resistance of insect pests to insecticides was noted long before the commercial introduction of DDT in the 1940's. According to Metcalfe (1955)- see also an article by the same author in Kogan (1986)- it was observed in 1914 in the San Jose Scale selected by lime sulfur spray. By 1946, resistance was observed in 11 species, including codling moth and various types of tick and thrip. Since that time, the growth in the number of cases of resistance to insecticides is, indeed, impressive, as Table 1 shows. There seems little doubt that, since at the time when the data in the table were collected the susceptibility of many insect pests had not been studied, this table provides an understatement of the true rate of proliferation of pesticide resistance. In addition, the development of cross resistance - the resistance of an insect to chemically related insecticides - and multiple resistance - resistance of a species to a wide variety of insecticides with differing modes of action - exacerbates the significance of the problem.

It is important to note that the problems of induced resistance are not specific to the controversial, because obviously toxic, insecticides such as DDT, lindane, carbaryl, and so on. Man's involvement in agriculture, whatever specific form it takes, will generally provoke biological responses, whether through mutagenic processes or simply through natural selection. For example, Martin and Woodcock (1983, p. 14) note that the introduction of rust-resistant varieties of wheat was, in time, followed by problems arising from the selection of strains of rust that were able to grow on the new varieties. One should also stress the importance of natural selection as a mechanism of resistance development. Martin and Woodcock (1983, p.234) state, in their discussion of the development of DDT-resistant strains of housefly, that "the process is one of selection, for DDT as ordinarily used has no mutagenic activity." The phasing out of, or reduced reliance on, toxic and carcinogenic insecticides, whatever other advantages it may bring, cannot be expected to solve the sorts of problems caused by the development of insect resistance to insecticides. Whatever physical, biological or chemical techniques are devised in our efforts to exploit and utilise the resources that nature has to offer, and to ensure that the fruits of such efforts are enjoyed by mankind rather than by competing species, we are likely to find ourselves in an arena in which our competitors will fight back. This is nothing more than the continuation of a biological "arms race" - see Dawkins and Krebs (1979) - which is as old as life itself, and indeed inseparable from it. The problem encountered in the use of antibiotics and of pesticides are, perhaps, simply dramatic examples of the phenomenon first suggested by van Valen (1973) and known to evolutionary biologists as the "Red Queen" theory- the Red Queen, it will be recalled, told Alice that "here...it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run twice as fast as that."

2.2 The Antibiotic Story

For a while, penicillin attained status similar to that of DDT as a potential "magic bullet." The story of its discovery by Alexander Flemming in 1928 is a famous anecdote in the annals of scientific discovery. During the following decade, Howard Florey and Ernest Chain were responsible for investigating further the properties of penicillin and for learning how to extract it, keep it stable, and produce it in large quatities. Florey also played a major role in persuading the United States government to support its large-scale production - a task that was greatly helped by its spectacularly successful use on victims of a nightclub fire in Boston in 1942. For a while, it seemed to some that antibiotics and related forms of chemotherapy would achieve the eradication of major killers such as tuberculosis, venereal diseases, septicemias, pneumonias, cholera, and many other diseases. According to Levy (1992), even in the usually hard-nosed medical literature of the time, it is possible to find extravagant claims for penicillin efficacy in treating cancers and viruses - claims now regarded as having no foundation. Readily available over the counter - prescriptions were not required until the mid-1950's - penicillin was widely touted as a panacea and taken both as therapeutic and prophylactic treatment. Among research laboratories, its remarkable successes encouraged the saerch for further antibiotics capable of treating other bacteria.

The penicillins, al though the best known, are only one of many groups of substances that have been developed and used as antibacterial agents. Many other naturally occuring antibacterial substances have been found and developed since the early 1940's for therapeutic use. Among the early discoveries were tetracyclines, which proved particularly effective against typhoid bacillus, and streptomycin, which was the first antibiotic able to kill the bacterium responsible for tuberculosis. In addition, many others are chemically synthesized. Sulfonamide derivatives, for example, were first synthesized in the 1930's and proved highly effective in controlling streptococcal infections. It is still the case that about 10% of all antimicrobial agents produced through the world are sulfonamides of one form or another.

By the 1970's, the experience had dampened the uncritical euphoria of the early 1940's, and a more balanced perspective on the capabilities and limitations of antibiotics emerged. It became clear to many of those involved that the success of any program with the ambitious task of substantially controlling, let alone exterminating, a major disease or pest depended on many things, involving not only biological and chemical science, but also political, social and, indeed, psychological factors. One problem that began to be encountered was the development of drug resistant strains of bacteria. This was not entirely unexpected. Levy (1992, p.7) quotes Alexander Flemming, one of the more prescient characters in this story, as warning the medical world as early as 1945 that "the greatest possibility of evil in self-medication is the use of too small doses so that instead of clearing up infection, the microbes are educated to resist penicillin and a host of penicillin-fast organisms is bred out which can be passed to other individuals and from them to others until they reach someone who gets a septicemia or a pneumonia which penicillin cannot save."

This passage is noteworthy for several reasons. First, it clearly recognises the way in which certain patterns of usage of antibiotics may promote the selection of resistant strains of bacteria. Second, it suggests a problem that economists would recognise as involving an externality - it is not merely the receiver of the antibiotic whose future defense against infection is jeopardised, but also other people. A third aspect, which we note but will be forced to ignore in our stylized formal modelling of the resulting externality problem, is the complexity of the mechanism involved. Flemming identifies too small a dosage level as a problem. Current scientific understanding suggests that, from a clinincal point of view, there is an optimum dosage, from which deviations both below and above may generate costs in terms of reduced efficacy of therapeutic substance on future occasions.

However, until the mid-1970's the drug-resistance problem was regarded, it seems, as little more than a minor irritant. Then two ominous developments shook medical experts. First, a strain of bacterium responsible for meningitis, and previously known to be particularly susceptible to ampicillin - a derivative of penicillin - was found that not only resisted, but *actually destroyed,* that antibiotic. At around the same time, it was found that strains of Neisseria gonorrheae, the bacterium responsible for gonorrheae, and hitherto successfully treated with penicillin, has emerged that were drug-resistant.

Since then, it has become very clear that the drug-resistance problem possesses a number of characteristics that make it an expensive and worrying phenomenon. The principal characteristics are: (i) the speed with which drug-resistant strains of bacteria have emerged in the wake of new antibiotics; (ii) their ability not only to resist but also to destroy antibiotics; (iii) the development of multiple drug-resistance; and (iv) the observation that most antibiotics encounter this range of problems, often within a short time of their initial introduction as therapeutic drugs. Moreover, it has become clear that the extend and speed of the development of drug resistance is influenced in important ways by patterns of usage. For this reason, the legal and economic frameworks within which drugs are administered, and the prevailing state of understanding and general attitudes to "appropriate" and "inappropriate" uses of drugs are all relevant in influencing the course of drug-resistance over time.

3 A Discrete Time Model

In this section we develop a three-period model. The advantages of this formulation is that, with a minimum of calculus, one can see the strategic interactions among firms in a dynamic context, without the need of acquiring a knowledge of more advanced techniques such as optimal control theory or dynamic programming. Also, the model is rich enough to generate multiple equilibria that can be Pareto ranked. After outlining the basic framework, we proceed to characterize the social optimum. This will be constrasted with the outcomes of the non-cooperative game, first under the assumption that firms use only open-loop strategies, then under the alternative assumption that they use Markov-perfect strategies[1].

There are n identical firms producing a homogenous good, whose price, denoted by $p > 0$ is exogenously given in the world market. The output of firm i at time t is q_{it}, where $t = 1, 2, 3$. The production function is

$$q_{it} = [A_t x_{it}]^{\beta}, \qquad 0 < \beta \leq 0.5$$

where $x_{it} \geq 0$ denote the input purchased by the firm i at the price w per unit, and A_t is the level of effectiveness of each unit of input. Let $N = \{1, 2, ..., n\}$.We assume that A_t declines over time, and its rate of decline at time t depends on the aggregate rate of use X_t where

$$X_t = \sum_{i \in N} x_{it} \tag{1}$$

Specifically, we assume that A_1 is given, and, for $t = 2, 3$,

$$A_t = \max\{0, A_{t-1} - X_{t-1}\} \tag{2}$$

[1]These concepts will be explained below in the context of our model. For more general expositions, see Clemhout and Wan (1994), Fudenberg and Tirole (1991), and Dockner et al. (2000).

Firm i's profit at time t is

$$\pi_{it} = p\left[A_t x_{it}\right]^{\beta} - w x_{it}$$

Note that in this model, the externality is not concurrent: output of firm j in period 1 affects firm i only in later periods, 2 and 3.

3.1 The social optimum

As a benchmark, let us assume in this section that there is a social planner who wants to maximize the discounted sum of the the profits of the n firms. Thus the planner chooses $x_i(t)$, $i \in N$, $t = 1, 2, 3$, to maximize

$$W = \sum_{t=1}^{3} \sum_{i \in N} \delta^t \pi_{it}$$

subject to (2) and (1), where $\delta \leq 1$ is the positive discount factor.

In what follows, for simplicity, we set $\delta = p = w = 1$. Since the production function is strictly concave in the input x_{it}, it is clear that the planner will choose $x_{it} = x_{jt}$ for all $j, i \in N$. We will therefore use the symbols x_t and π_t to denote the input level and the profit of the representative firm at time t. The planner's problem is thus to choose $x_t, t = 1, 2, 3$, to maximize

$$W = \sum_{t=1}^{3} \left[n\left[A_t x_t\right]^{\beta} - n x_t\right]$$

subject to

$$A_t = \max\left\{0, A_{t-1} - n x_{t-1}\right\}$$

We solve this problem by the backward solution method. While numerical solutions can be found for any β, for analytical purposes, we prefer to obtain a closed-form solution, and to do so, it is convenient to set $\beta = 1/2$. At the beginning of perod 3, given A_3, the maximization of $n\pi_3$ yields

$$x_3^* = \frac{1}{4} A_3$$

and hence

$$\pi_3^* = \frac{1}{4} A_3 = \frac{1}{4} \max\left\{0, A_2 - n x_2\right\}$$

Turning to period 2, given A_2, the objective is to find x_2 that maximize $n\pi_2 + n\pi_3^*$. The solution is

$$x_2^* = D_2 A_2$$

where

$$D_2 = \frac{1}{4}\left(\frac{1}{1+\frac{n}{4}}\right)^2 = \left[\frac{2}{4+n}\right]^2 < \frac{1}{n}$$

Hence

$$\pi_2^* + \pi_3^* = B_2 A_2 = B_2 \max\{0, A_1 - n x_1\}$$

where

$$B_2 = \frac{n}{4(4+n)}$$

Finally, in period 1, one seeks to maximize $n\pi_1 + n(\pi_2^* + \pi_3^*)$. This yields

$$x_1^* = D_1 A_1$$

where

$$D_1 = \frac{1}{4}\left(\frac{1}{\frac{n}{4} + \left(1+\frac{n}{4}\right)^2}\right)^2 = \left[\frac{8+2n}{16+12n+n^2}\right]^2 < \frac{1}{n}$$

The optimal value of the objective function is

$$W = nA_1\left[D_1 - D_1^{1/2} + B_2(1 - nD_1)\right]$$

and A_2^* and A_3^* are both strictly positive. (It can be verified that any policy that drives A_2 or A_3 to zero will yield a lower welfare.)

3.2 Open loop Nash equilibria, or equilibria with precommitment

We now consider a game among the n firms. In this subsection, we restrict attention to open loop Nash equilibria (OLNE), also called equilibria with precommitment. That is, we allow firms to have only open-loop (also called precommitment) strategies. Given A_1, an open-loop strategy of firm i is a triple $(x_{i1}, x_{i2}, x_{i3}) \equiv s_i$ that represents the planned path of input levels for the three periods. It is important to note that, by definition of an open-loop strategy, the future input levels x_{i2} and x_{i3} must not be made conditional on the observed stock levels A_2 and A_3. (It is as though firms would not be able to observe A_2 and A_3.) Each firm must precommit itself to a strategy, from the outset. Firm j takes the strategies s_i $(i \neq j)$ as given, and chooses (x_{j1}, x_{j2}, x_{j3}) to maximize

$$\Pi_j = \sum_{t=1}^{3} \pi_{jt} \tag{3}$$

subject to

$$A_{t+1} = \max\left\{0, A_t - X_t^{-j} - x_{jt}\right\} \qquad t = 1, 2, \qquad A_1\text{=given}, \tag{4}$$

where

$$X_t^{-j} \equiv \sum_{i \neq j} x_{it}$$

(obtained from the strategies s_i , $i \neq j$) is taken as given. The solution of this problem yields $(x_{j1}^*, x_{j2}^*, x_{j3}^*) \equiv s_j^*$ as a best reply to the strategies s_i $(i \neq j)$.

An OLNE is defined as a strategy profile $(\widehat{s}_1, \widehat{s}_2, \widehat{s}_3, ..., \widehat{s}_n)$ such that for each j, $\widehat{s}_j$ is a best reply to the strategies $(i \neq j)$. In what follows, we restrict attention to symmetric OLNE, where all firms use the same strategy in equilibrium. Notice that each firm, knowing the equilibrium strategies, can predict correctly (at least in the case of unique equilibrium) the equilibrium values of A_2 and A_3. By definition of a precommitment strategy, if the observed values of A_2 and A_3 for some reason turn out to be different from the predicted values, the firm will not change its committed levels of inputs x_{j2}^*, x_{j3}^*. Clearly, such precommitment may make sense if all firms believe that by having every players making binding promises to follow their strategy to the end, there will be no deviation of the observed values of A_2 and A_3 from their equilibrium values, and that this is to their advantage. (Implicit in the concept of precommitment equilibrium is an enforcement mechanism, and an enforcement authority. For example, duelists may agree not to carry extra weapons, two boxers may agree not to kick, and one could expect such precommitment to be honoured if the penalty for violation is sufficiently great.)

To find a symmetric OLNE, we begin by solving problem (3) for firm j. It is convenient to use the backward solution method. In period 3, given A_3, since there is no future ahead, the optimal x_{j3} is

$$x_{j3}^* = \frac{1}{4} A_3$$

and

$$\pi_{j3}^* = \frac{1}{4}A_3$$

In period 2, given $A_2 \geq 0$ and $X_2^{-j} \geq 0$, firm j seeks to maximize

$$[x_{j2}A_2]^{1/2} - x_{j2} + \frac{1}{4}\max\left\{0, A_2 - X_2^{-j} - x_{j2}\right\}$$

The solution depends on whether the term $A_2 - X_2^{-j} - x_{j2}^*$ is positive or not. If this term is zero or negative, then $A_3 = 0$ and period 3 becomes irrelevant, and hence

$$x_{j2}^* = \frac{1}{4}A_2 \text{ and } \pi_{j2}^* + \pi_{j3}^* = \frac{1}{4}A_2 \tag{5}$$

If $A_2 - X_2^{-j} - x_{j2}^*$ is positive, then

$$x_{j2}^* = \frac{4}{25}A_2 \text{ and } \pi_{j2}^* + \pi_{j3}^* = \frac{9}{25}A_2 - \frac{1}{4}X_2^{-j}$$

(If $X_2^{-j} \geq (21/25)A_2$, then $x_{j2}^* = \frac{4}{25}A_2$ is not a solution, because if it were a solution, it would give $A_3 = 0$, yielding $\pi_{j2}^* + \pi_{j3}^* < (1/4)A_2$; it follows that if $n \geq 7$, there can be no symmetric equilibrium with $x_{j2}^* = \frac{4}{25}A_2 > 0$.)

In period 1, given A_1 and X_1^{-j}, firm j seeks to maximize

$$[x_{j1}A_1]^{1/2} - x_{j1} + \pi_{j2}^* + \pi_{j3}^*$$

The solution depends on the sign of $A_1 - X_1^{-j}$ and the sign of $A_1 - X_1^{-j} - X_2^{-j}$:

(a) If $A_1 - X_1^{-j} \leq 0$, then periods 2 and 3 become irrelevant, and hence $x_{j1}^* = (1/4)A_1 = \pi_{j1}^* + \pi_{j2}^* + \pi_{j3}^*$.

(b) If $A_1 - X_1^{-j} > 0$ but $A_1 - X_1^{-j} - X_2^{-j} \leq 0$ then period 3 becomes irrelevant, and hence firm j maximizes

$$[x_{j1}A_1]^{1/2} - x_{j1} + \frac{1}{4}\max\left\{0, A_1 - X_1^{-j} - x_{j1}\right\}$$

we thus have

$$x_{j1}^* = \frac{4}{25}A_1 \text{ and } \pi_{j1}^* + \pi_{j2}^* + \pi_{j3}^* = \frac{9}{25}A_1 - \frac{1}{4}X_1^{-j} + 0$$

(c) If $A_1 - X_1^{-j} > 0$ and $A_1 - X_1^{-j} - X_2^{-j} > 0$, then firm j maximizes

$$[x_{j1}A_1]^{1/2} - x_{j1} + \frac{1}{4}A_2 \tag{6}$$

if it chooses A_3 to be zero. In this case, it obtains

$$x_{j1}^* = \frac{4}{25}A_1 \text{ and } \Pi_j = \frac{9}{25}A_1 - \frac{1}{4}X_1^{-j}$$

(d) On the other hand, if $A_1 - X_1^{-j} > 0$ and $A_1 - X_1^{-j} - X_2^{-j} > 0$, and if the firm finds its optimal to have $A_3 > 0$, then the solution is obtained from maximizing the expression

$$(x_{j1}A_1)^{1/2} + \left[x_{j2}\left(A_1 - x_{j1} - X_1^{-j}\right)\right]^{1/2} + \left[x_{j3}\left(A_1 - x_{j1} - x_{j3} - X_1^{-j} - X_2^{-j}\right)\right]^{1/2} - \sum_{t=1}^{3} x_{jt} \tag{7}$$

First order conditions for (7) with respect to x_{j3},x_{j2} and x_{j3}, in that order, are

$$\left[\frac{A_1 - x_{j1} - x_{j3} - X_1^{-j} - X_2^{-j}}{x_{j3}}\right]^{1/2} = 2$$

$$\left[\frac{A_1 - x_{j1} - X_1^{-j}}{x_{j2}}\right]^{1/2} - \left[\frac{A_1 - x_{j1} - x_{j3} - X_1^{-j} - X_2^{-j}}{x_{j3}}\right]^{-1/2} = 2$$

and

$$\left[\frac{A_1}{x_{j1}}\right]^{1/2} - \left[\frac{A_1 - x_{j1} - X_1^{-j}}{x_{j2}}\right]^{-1/2} - \left[\frac{A_1 - x_{j1} - x_{j3} - X_1^{-j} - X_2^{-j}}{x_{j3}}\right]^{-1/2} = 2$$

from which

$$x_{j3} = \frac{1}{4}\left(A_1 - x_{j1} - x_{j3} - X_1^{-j} - X_2^{-j}\right)$$

$$x_{j2} = \frac{4}{25}\left(A_1 - x_{j1} - X_1^{-j}\right)$$

and

$$x_{j1} = A_1\left(\frac{10}{29}\right)^2 = 0.1189A_1$$

Define an *interior symmetric OLNE* as a symmetric OLNE with $x_{jt} > 0$ for all $j \in N$ and for all $t = 1, 2, 3$. From the above observations, we obtain the following proposition.

Proposition 1: (interior symmetric OLNE)

An interior symmetric OLNE exists if and only if $n \leq 6$. The equilibrium open-loop strategy of the representative firm j is

$$x_{j1} = \left(\frac{10}{29}\right)^2 A_1 > 0$$

$$x_{j2} = \frac{4}{25}\left[1 - n\left(\frac{10}{29}\right)^2\right] A_1 > 0$$

$$x_{j3} = \frac{1}{4}\left[1 - \frac{4n}{25}\right]\left[1 - n\left(\frac{10}{29}\right)^2\right] A_1 > 0$$

Along the resulting equilibrium path, the state variable A_t evolves as follows

$$A_2 = \left[1 - n\left(\frac{10}{29}\right)^2\right] A_1 > 0$$

and

$$A_3 = \left[1 - \frac{4n}{25}\right] A_2 > 0.$$

Proof: omitted.

Proposition 2: (non-interior symmetric OLNE, with positive input in period one only)

For $n \geq 4$, the following is an OLNE:

$$x_{j1} = \frac{1}{4} A_1 > 0$$

$$x_{j2} = x_{j3} = 0$$

Along the resulting equilibrium path, the state variable A_t evolves as follows: $A_2 = A_3 = 0$.

Proof: obvious.

Proposition 3: (non-interior symmetric OLNE, with positive inputs in periods one and two only)

For $n = 5$ and $n = 6$, there is a third symmetric OLNE, with

$$x_{j1} = \frac{4}{25} A_1 > 0$$

and

$$x_{j2} = \frac{1}{4}\left[1 - \frac{4n}{25}\right] A_1 > 0, \qquad x_{j3} = 0.$$

Along the resulting equilibrium path, the state variable A_t evolves as follows

$$A_2 = \left[1 - \frac{4n}{25}\right] A_1 > 0, \qquad A_3 = 0.$$

Proof: obvious.

Remark: Propositions 1, 2 and 3 imply that there are multiple symmetric equilibria for $n = 4, 5, 6$. These equilibria are Pareto rankable. For example, if $n = 5$, then the equilibria in propositions 1,2, and 3 give total profit $\Pi_j = 0.343A_1$, $\Pi_j = 0.328A_1$ and $\Pi_j = 0.250A_1$ respectively.

3.3 Markov-perfect Nash equilibria, or equilibria without precommitment

There is an obvious weakness in the concept of OLNE. If, indeed, we were to assume that all input decisions have to be made at the outset, then such an equilibrium would make a good deal of sense. It results in an allocation such that, *after the event, no player has reason to regret his choice.* Each, on reflection, can be imagined as thinking,"given everyone else's strategy led to X_1^{*-j} and X_2^{*-j}, my chosen strategy $(x_{j1}^*, x_{j2}^*, x_{j3}^*)$ was my most profitable. If I were to go through this exercise again, I would not therefore have any incentive to behave otherwise." Even in a model of sequential choices, this argument may continue to make sense if the realized values of A_2 and A_3 cannot be observed until after all choices are made. However, if A_t $(t = 2, 3)$ can be observed before the decision x_{jt} $(t = 2, 3)$ are made, then it makes sense to allow j to condition his input level on the observed stock value. If j is able to do this, it makes sense to allow j to acknowledge that other players can, and rationally will, do so.

The above argument leads us to turn to equilibria with feedback, also called Markov perfect Nash equilibria (MPNE). We look for strategies that condition each period input level on the current level of effectiveness A_t. A feedback strategy is a triple $(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) \equiv \sigma_j$ where each $x_{jt}(.)$ is a *rule* that prescribes firm j's input level for period t in response to the observed effectiveness level A_t. A MPNE is defined as a strategy profile $(\widehat{\sigma}_1, \widehat{\sigma}_2, \widehat{\sigma}_3, ..., \widehat{\sigma}_n)$ such that for each j, $\widehat{\sigma}_j$ is a *sub-game perfect best reply* to the strategies $\widehat{\sigma}_i (i \neq j)$. *Sub-game perfectness* means that, at any time t $(t = 1, 2, 3)$ and given any non-negative number A_t, it is in the best interest of firm j to continue using (the remaining part of) its strategy $\widehat{\sigma}_j$, given that other firms continue using (the remaining part of) their strategies $\widehat{\sigma}_i (i \neq j)$.

In what follows, we restrict attention to symmetric MPNEs, where all firms use the same strategy in equilibrium.

Firm j takes the strategies σ_i $(i \neq j)$ as given, and chooses the rules $(x_{j1}(.), x_{j2}(.), x_{j3}(.))$ to maximize

$$\Pi_j = \sum_{t=1}^{3} \pi_{jt}$$

subject to

$$A_{t+1} = \max\left\{0, A_t - X_t^{-j}(A_t) - x_{jt}\right\} \qquad t = 1, 2, \qquad A_1\text{=given},$$

where

$$X_t^{-j}(A_t) \equiv \sum_{i \neq j} x_{it}(A_t).$$

Again, we solve this problem backwards, starting from period 3.

In period 3, given A_3,the optimal choice for firm j is simple, because this is the last period, there is no future to worry about. Thus firm j has the dominant choice:

$$x_{j3}(A_3) = \frac{1}{4} A_3 \tag{8}$$

yielding the profit

$$\pi_{j3}^*(A_3) = \frac{1}{4} A_3 \equiv V_{j3}^*(A_3)$$

Turning to period 2, given A_2, the objective is to maximize

$$V_2 = (x_{j2} A_2)^{1/2} - x_{j2} + V_{j3}^*(A_3)$$

where

$$A_3 = \min\left\{0, A_2 - x_{j2} - X_2^{-j}(A_2)\right\}$$

It is convenient to treat separately two possibilities, according to whether the outcome A_3 that would result from $X_2^{-j}(A_2)$ and from the choice of x_{j2} is positive or zero. If the outcome is $A_3 = 0$, then period 2 is effectively the last period, and it must be the case that

$$x_{j2}(A_2) = \frac{1}{4} A_2 \tag{9}$$

and thus

$$\pi_{j2} + \pi_{j3} = \frac{1}{4} A_2. \tag{10}$$

This outcome is possible only if $n \geq 4$ (for otherwise (9) would not result in $A_3 = 0$).

Alternatively, $A_3 > 0$, in which case firm j's optimal input level x_{j2} must maximize

$$(x_{j2}A_2)^{1/2} - x_{j2} + \frac{1}{4}\left[A_2 - x_{j2} - X_2^{-j}(A_2)\right]$$

yielding

$$x_{j2}(A_2) = \frac{4}{25}A_2 \tag{11}$$

Clearly, this is consistent with a symmetric equilibrium with $A_3 > 0$ only if $n \leq 6$.

For $n > 6$, we cannot have $x_{j2}(A_2) = \frac{4}{25}A_2$ as a best reply for $x_{i2}(A_2) = \frac{4}{25}A_2$ ($i \neq j$) for it would result in $A_3 = 0$ and hence $\pi_{j2} + \pi_{j3} = \frac{4}{25}A_2$, which is inferior to (10).

It remains to consider the case $n \leq 6$. In this case there are two candidate rules for period 2: namely (a) rule (11) for all j, and (b)rule (9) for all j. To show that (a) constitutes an equilibrium for period 2 (given that in period 3, eveyone will choose the rule (8)), we must show that given that all $j \neq k$ choose rule (11), then firm k has no interest to deviate from it. It turns out out that deviation is not optimal for k in the case $n \leq 6$. For (b), matters are not so simple. It can be verified that for $n = 5$ and also for $n = 6$, if every $j \neq k$ uses rule (9), then k will use rule (9), too. For $n = 4, 3$,or 2, if every $j \neq k$ uses rule (9), then k will use rule (11) instead.

Thus we have obtained the following proposition for all the subgames starting with $A_2 > 0$:

Proposition 4: (symmetric MPNE starting in period 2)

Given any $A_2 > 0$,

(i) if $n \geq 7$, then the only symmetric MPNE is the strategy $(x_{j2}(A_2)$,$x_{j3}(A_3)) = \left[\frac{1}{4}A_2, \frac{1}{4}A_3\right]$,resulting in $A_3 = 0$ and the resulting profit is

$$\pi_{j2} + \pi_{j3} = \frac{1}{4}A_2$$

(ii) if $n \leq 4$, then the only symmetric MPNE is the strategy $(x_{j2}(A_2)$,$x_{j3}(A_3)) = \left[\frac{4}{25}A_2, \frac{1}{4}A_3\right]$, resulting in $A_3 > 0$ and the resulting profit is

$$\pi_{j2} + \pi_{j3} = \left[\frac{49 - 4n}{100}\right] A_2$$

(iii) if $n = 5$ or $n = 6$, then there are *two* symmetric MPNEs :

$$\text{(a) } (x_{j2}(A_2), x_{j3}(A_3)) = \left[\frac{1}{4}A_2, \frac{1}{4}A_3\right]$$

which yields $A_3 = 0$ and $\pi_{j2} + \pi_{j3} = \frac{1}{4}A_2$,

$$\text{(b) } (x_{j2}(A_2), x_{j3}(A_3)) = \left[\frac{4}{25}A_2, \frac{1}{4}A_3\right]$$

which yields $A_3 > 0$ and $\pi_{j2} + \pi_{j3} = \frac{29}{100}A_2$ if $n = 5$, and $\pi_{j2} + \pi_{j3} = \frac{25}{100}A_2$ if $n = 6$.

Proof: omitted.

We are now ready to tackle the problem of period 1. In view of proposition 4, we need to consider three cases:

Case A: $n \leq 4$

Case B: $n \geq 7$

Case C: $n = 5$ or 6.

CASE A: $n \leq 4$

Firm j's problem in period 1 is to find x_{j1} that maximizes

$$(x_{j1}A_1)^{1/2} - x_{j1} + \left(\frac{49 - 4n}{100}\right) \max\left\{0, A_1 - x_{j1} - X_1^{-j}(A_1)\right\}$$

given A_1 and given $X_1^{-j}(A_1)$. Again, there are two possibilities in a symmetric equilibrium. Either

(i) $A_1 - x_{j1} - X_1^{-j}(A_1) > 0$, implying $A_2 > 0$, or

(ii) $A_1 - x_{j1} - X_1^{-j}(A_1) \leq 0$, implying $A_2 = 0$.

Under (i), the first order condition yields

$$x_{j1}(A_1) = \left(\frac{50}{149 - 4n}\right)^2 A_1 \qquad (12)$$

while under (ii), the first order condition yields

$$x_{j1}(A_1) = \frac{1}{4}A_1 \qquad (13)$$

which, in Case A, is consistent $A_2 = 0$ only if $n = 4$ (given symmetry in the behavior of firms). However, it can be verified that (13) is not consistent with a symmetric *equilibrium* for $n = 4$: if all other firms choose (13), then firm j can deviate by choosing (12) and, thus ensuring that $A_2 > 0$, and earning a higher value for $\pi_{j1} + \pi_{j2} + \pi_{j3}$, amounting to

$$\left(\frac{50}{149-4n}\right) A_1 - \left(\frac{50}{149-4n}\right)^2 A_1 + \left(\frac{49-4n}{100}\right)\left[1 - \frac{n-1}{4} - \left(\frac{50}{149-4n}\right)^2\right] A_1$$

which exceeds the profit it would earn by following (13), which is $\pi_{j1} + \pi_{j2} + \pi_{j3} = \frac{1}{4}A_1 + 0 + 0$.

Proposition 5A: (symmetric MPNE starting in period 1, when $n \leq 4$)

For $n \leq 4$, the only symmetric MPNE is

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{\left(\frac{50}{149-4n}\right)^2 A_1, \left(\frac{4}{25}\right) A_2, \left(\frac{1}{4}\right) A_3\right\}$$

CASE B: $n \geq 7$

In this case, it is clear that $x_{j1}(A_1) = \frac{1}{4}A_1$ for all j yields a symmetric MPNE, resulting in $A_2 = 0$. Does there exist other symmetric MPNEs ? In view of part (i) of proposition 4, firm j's profit is

$$(x_{j1}A_1)^{1/2} - x_{j1} + \left(\frac{1}{4}\right) \max\left\{0, A_1 - x_{j1} - X_1^{-j}(A_1)\right\}$$

Now either

(i) $A_1 - x_{j1} - X_1^{-j}(A_1) > 0$, implying $A_2 > 0$, or

(ii) $A_1 - x_{j1} - X_1^{-j}(A_1) \leq 0$, implying $A_2 = 0$.

Under (i) the first order condition gives $x_{j1} = \frac{4}{25}A_1$, which is inconsistent with $A_2 > 0$, since $n \geq 7$. So the only possibility is (ii), under which the first order condition gives $x_{j1}(A_1) = \frac{1}{4}A_1$. Therefore, we obtain

Proposition 5B: (symmetric MPNE starting in period 1, when $n \geq 7$)

For $n \geq 7$, the only symmetric MPNE is

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{\left(\frac{1}{4}\right) A_1, \left(\frac{1}{4}\right) A_2, \left(\frac{1}{4}\right) A_3\right\}$$

and along the equilibrium play, $A_2 = A_3 = 0$.

CASE C: $n = 5$ or $n = 6$.

Applying an argument similar to the one used to establish propositions 5A and 5B, we obtain

Proposition 5C: (symmetric MPNE starting in period 1, when $n = 5$ or $n = 6$)

(i) For $n = 6$, there are three symmetric MPNEs, all yielding the same payoff ($= \frac{1}{4}A_1$). They are

Equilibrium #1:

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{ \left(\frac{50}{149 - 4n}\right)^2 A_1, \left(\frac{4}{25}\right) A_2, \left(\frac{1}{4}\right) A_3 \right\}$$

implying positive inputs x_{jt} for all $t = 1, 2, 3$.(Since A_2 and A_3 are positive when firms use this equilibrium profile of strategies.)

Equilibrium #2:

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{ \left(\frac{1}{4}\right) A_1, \left(\frac{4}{25}\right) A_2, \left(\frac{1}{4}\right) A_3 \right\}$$

implying positive inputs only in period1.(Since $A_3 = A_2 = 0$ when firms use this equilibrium profile of strategies.)

Equilibrium #3:

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{ \left(\frac{1}{4}\right) A_1, \left(\frac{1}{4}\right) A_2, \left(\frac{1}{4}\right) A_3 \right\}$$

implying positive inputs only in period1.(Since $A_3 = A_2 = 0$ when firms use this equilibrium profile of strategies.)

(i) For $n = 5$, there are four symmetric MPNEs, yielding different pay-offs.

Equilibrium #1:

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{ \left(\frac{50}{149 - 4n}\right)^2 A_1, \left(\frac{4}{25}\right) A_2, \left(\frac{1}{4}\right) A_3 \right\}$$

implying positive inputs in all three periods1.(Since A_2 and A_3 are positive when firms use this equilibrium profile of strategies.) The pay-off is $0.3095A_1$.

Equilibrium #2:

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{ \left(\frac{4}{25}\right) A_1, \left(\frac{1}{4}\right) A_2, \left(\frac{1}{4}\right) A_3 \right\}$$

implying positive inputs in periods 1 and 2.(Since $A_3 = 0$ when firms use this equilibrium profile of strategies.) The pay-off is $0.25A_1$

Equilibrium #3:

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{ \left(\frac{1}{4}\right) A_1, \left(\frac{4}{25}\right) A_2, \left(\frac{1}{4}\right) A_3 \right\}$$

implying positive inputs only in period1.(Since $A_3 = A_2 = 0$ when firms use this equilibrium profile of strategies.) The pay-off is $0.25A_1$

Equilibrium #4:

$$(x_{j1}(A_1), x_{j2}(A_2), x_{j3}(A_3)) = \left\{ \left(\frac{1}{4}\right) A_1, \left(\frac{1}{4}\right) A_2, \left(\frac{1}{4}\right) A_3 \right\}$$

implying positive inputs only in period1.(Since $A_3 = A_2 = 0$ when firms use this equilibrium profile of strategies.) The pay-off is $0.25A_1$

Remark: The most notable feature that arises when $n = 5$ is that, not only are there multiple equilibria, but one equilibrium Pareto dominates the others.

4 A Continuous Time Model

We now turn to a continuous time model with an infinite horizon. The advantage of this approach is that a diagrammatic representation of equilibria can be obtained. Furthermore, it avoids the problem of an abrupt end which arises when a finite horizon is imposed.

There are n firms using a pesticide as an input. Let $N \equiv \{1, 2, ..., n\}$. Again, let $A(t)$ denote the effectiveness of the pesticide at time t, and $x_i(t)$ be the dosage level applied by firm i at time t. We assume that at any moment of time, the higher the current aggregate dosage, the greater the decline in effectiveness. Specifically, we assume the relationship takes the simple linear form

$$\dot{A}(t) = -b \sum_{i \in N} x_i(t). \tag{14}$$

The firms produce a homogenous good. Output of firm i at time t is denoted by $Q_i(t)$ and is determined by a production function of the form

$$Q_i(t) = (A(t)x_i(t))^{\alpha}$$

and firm i's profit is

$$\pi_i(t) = p(A(t)x_i(t))^{\alpha} - cx_i(t)$$

where p, the price of the good, and c, the cost of of a dose of pesticide, are assumed to be exogenously determined.

4.1 The social optimum

Suppose there is a social planner who wishes to maximize

$$W = \int_0^{\infty} \sum_{i \in N} \pi_i(t) e^{-rt} dt$$

where $r > 0$ is the rate of discount. The maximization is subject to (14) and the boundary conditions

$$A(0) = A_0 \tag{15}$$

$$\lim_{t \to \infty} A(t) \geq 0. \tag{16}$$

This is an optimal control problem[2] that resembles the problem of optimal extraction of a non-renewable resource[3]. A major difference is that in the present model, the state variable A appears in the production function in a multiplicative form. For a closed form solution, we set $c = 0$ in what follows. (If $c > 0$ then numerical and/or qualitative solution methods must be used.)

The following proposition describes the solution of the social planner's problem.

Proposition 6: The optimal time path of $A(t)$ is

$$A(t) = A_0 \exp\left[\frac{-rt}{2(1-\alpha)}\right] \tag{17}$$

and the optimal doses are

$$nbx_j(t) = \frac{rA(t)}{2(1-\alpha)}$$

Thus both $A(t)$and $x_j(t)$ are monotone decreasing and approach zero asymptotically. The value of discounted aggregate profit flow of is

$$W = KA_0^{2\alpha} \tag{18}$$

where

$$K = p\left[\frac{n(1-\alpha)}{r}\right]^{1-\alpha} \left(\frac{1}{2b}\right)^{\alpha}$$

[2]See Leonard and Long (1992) for a treatment of optimal control theory with economic applications.

[3]See, for example, Kemp and Long (1980).

Proof: omitted. (Verify that the necessary and sufficient conditions for the general infinite horizon optimal control problem, as stated in Leonard and Long (1992, Chapter 9) are satisfied, when the shadow price is $\psi(t) = 2\alpha A(t)^{2\alpha-1}$.)

4.2 Markov perfect Nash equilibria

In this section, we find Markov perfect Nash equilibria for the game among the n firms. We restrict attention to stationary Markov strategies. We define a stationary Markov strategy for player j as a function $\phi_j(.)$ which specifies the input level $x_j(t)$ as a function of the currently observed value of A, i.e., of $A(t)$:

$$x_j(t) = \phi_j(A(t)).$$

In our model, the influence of the past at any time t is summed up in the value of the state varible $A(t)$. Our formulation is restrictive in that we do not allow a firm to threaten to punish other firms on the basis of the history of their actions. (Formally, this is as if each firm could only observe the current value $A(t)$ and had no memories of past actions of other firms, nor of the history of A.) We believe that our restriction is reasonable, because if the strategy space is enlarged, it is likely that a result similar to the folk theorem[4] in supergames will apply: namely, the cooperative outcome can be supported by suitable threat strategies, if the rate of discount is not too high. The fact that cooperative outcomes do not seem happen in practice seems to indicate that theorists are sometimes too generous in endowing firms with a large set of strategies to choose from.

We define a Markov perfect Nash equilibrium (MPNE) as a profile of stationary Markov strategies $(\phi_1^*, \phi_2^*, .., \phi_n^*)$ such that such that for any firm j, the following inequality holds for all initial state A_0 and all stationary Markov strategies $\phi_j(.)$:

$$J_j(\phi_j^*, \phi_{-j}^*, A_0) \geq J_j(\phi_j, \phi_{-j}^*, A_0)$$

where ϕ_{-j}^* is the vector of strategies of all firms other than j, and where

$$J_j(\phi_j, \phi_{-j}^*, A_0) = \int_0^\infty p\left[A(t)\phi_j(A(t))\right]^\alpha \exp(-rt)dt$$

[4] See, for example, Fudenberg and Tirole (1991).

subject to (15),(16) and

$$\dot{A}(t) = -b\phi_j(A(t)) - b\sum_{i\neq j}\phi_i^*(A(t)).$$

Analysis of this problem leads to the following proposition.

Proposition 7:

(i) If $1-n\alpha > 0$, then there exists a MPNE where the equilibrium strategy of each firm is a linear function of the stock

$$x_j(t) = \frac{rA(t)}{2b(1-n\alpha)} \equiv \phi_j^*(A(t))$$

and the resulting time path of the state variable is

$$A(t) = A_0 \exp\left[\frac{-rnt}{2(1-n\alpha)}\right] \tag{19}$$

so that $A(t)$ tends to zero only asymptotically. The value of the integral of discounted profit of each firm is

$$J_j(A_0) = kA_0^{2\alpha} \tag{20}$$

where

$$k \equiv p\left[\frac{1-n\alpha}{r}\right]^{1-\alpha}\left(\frac{1}{2b}\right)^{\alpha}$$

(ii) If $1-n\alpha > 0$ and $n \geq 2$, then in addition to the the linear stationary Markov strategy as described above, there are other MPNEs in which non-linear stationary Markov strategies are played. These equilibria lead to the vanishing of $A(t)$ in finite time.

(iii) If $1-n\alpha < 0$, then the only "equilibrium" is that all firms try to exhaust A_0 at the first instant.

Proof: Omitted. See Appendix 1.

Remark: Comparing proposition 7 with proposition 6, we see that the effectiveness of the pesticide declines too fast when there is lack of cooperation.

4.3 Open-loop Nash equilibria

An open-loop strategy must specify the time path of input x_j as function of time t and of parameters of the model, such as p,n,α,r, and A_0. (Note that A_0 is treated as a parameter, because it is given, while $A(t)$ is not a parameter, for $t > 0$.) Formally, we define an open-loop strategy for player i as a piece-wise continuous function $g_j(.)$ that specifies for each $t \geq 0$ a value $x_j(t) \geq 0$. An open-loop Nash equilibrium is a profile of strategies $(g_1^*, g_2^*, ..., g_n^*)$ such that for all player $j \in N$, and for all open-loop strategies g_j, the following inequality holds

$$J_j(g_j^*, g_{-j}^*) \geq J_j(g_j, g_{-j}^*)$$

where $g_{-j}^* \equiv (g_1^*, g_2^*, ..., g_{j-1}^*, g_{j+1}^*, ..., g_n^*)$ and where

$$J_j(g_j, g_{-j}^*) \equiv \int_0^\infty p\left[A(t)g_j(t)\right]^\alpha \exp(-rt)dt$$

such that

$$A(0) = A_0$$

$$\lim_{t\to\infty} A(t) \geq 0$$

and

$$\dot{A}(t) = -bg_j(t) - b\sum_{i\neq j} g_i^*(t).$$

The following proposition can be proved:

Proposition 8:

(i) If $1-n\alpha > 0$, then there exists a symmetric OLNE where the open-loop strategy of the representative firm j satisfies:

$$bx_j(t) = \left[\frac{r}{2(1-n\alpha)+(n-1)}\right] A_0 \exp\left[\frac{-nrt}{2(1-n\alpha)+(n-1)}\right] \equiv bg_j^*(t) \tag{21}$$

and the resulting time path of the state variable is

$$A(t) = A_0 \exp\left[\frac{-nrt}{2(1-n\alpha)+(n-1)}\right] \tag{22}$$

(ii) If $1 - n\alpha > 0$ and $n > 2$, there exists other OLNEs with finite exhaustion time.

(iii) If $1 - n\alpha < 0$ then the only "equilibrium" is that all firms try to exhaust A_0 at the first instant.

Proof: Omitted. (It is similar to the proof of proposition 7.)

Remark:From (21) and (22), we can express the *outcome* of the OLNE in the *feedback form:*

$$nbx_j(t) = \left[\frac{nr}{2(1-n\alpha)+(n-1)}\right]A(t) \tag{23}$$

which is not to be confused with a Markov strategy.

Comparing part (i) proposition 8 with that of proposition 7 and with proposition 6, we see that while in all three cases the effectiveness declines to zero only asymptotically, lack of cooperation results in faster rates of decline than the one which is socially optimal. In addition, comparing MPNE and OLNE, we notice that in a MPNE $A(t)$ declines faster than in an OLNE. This is intuitively plausible, since in a MPNE each player knows that other players will use less input if the effectiveness is low. Therefore each wants to use more input, so as to reduce the future input levels of other, thus having more for himself.This is similar to a Cournot game with negative conjectural variations: each firm thinks that if it were to produce more, the other will reduce output, and as a result of that reasoning, they are further away from the cooperative outcome.

5 Concluding Remarks

There is a large literature that investigates implications of positive externalities flowing from current productive activities to enhanced future productivity. This paper addresses the opposite situation, in which the present applications of inputs lowers the effectiveness of future doses of those same inputs. It seems clear that antibiotics and pesticides are such inputs.

We have not addressed the question of optimal regulation. It is possible to achieve the social optimum by designing a tax rule that determines the tax rate on input use at any time as the function of the level of the state variable at that time. (See Benchekroun and Long (1998) for such a scheme, in the context of a polluting oligopoly.) Another issue is the optimal usage of the existing drug when a new drug is expected to become available in the future. We have also abstracted from the population dynamics of pests and bacteria. These are interesting topics for future research.

The analysis of this paper has focused on the case where the same individuals or firms that make decisions today will have to suffer from the consequences of such decisions. An issue which we have neglected is that quite often the later adverse effects may be borne by individuals other than those responsible for the initial decision to apply the input, thereby constituting a classic externality problem. It is not clear to us how significant these sources of inefficiency may be, for the simple reason that they have, so far, attracted little interest. We hope that this paper will, if nothing else, stimulate others to give these issues more serious and sustained attention than they have hitherto received.

APPENDIX

Markov-Perfect Nash Equilibrium

The Hamilton-Jacobi-Bellman equation for firm j is

$$rV_j(A) = \max_{x_j} \left\{ p(Ax_j)^\alpha - bV_j'(A)\left[x_j + (n-1)\phi_i(A)\right] \right\} \tag{A1}$$

This yields the first order condition

$$\alpha p A^\alpha x_j^{\alpha-1} = bV_j'(A) \tag{A2}$$

From (A2) and the assumption of symmetry, (A1) becomes

$$rV_j(A) = p\left[A\phi(A)\right]^\alpha (1 - n\alpha) \tag{A3}$$

Since each firm can always ensure that $V_j(A) \geq 0$, it follows that if $1 - n\alpha < 0$ then (A3) cannot be satisfied. Thus no (non-degenerate) MPNE exists if $1 - n\alpha < 0$.

Consider now the case $1 - n\alpha > 0$. Let $x(A) \equiv \phi(A)$. Differentiate (A3) with respect to A

$$rV_j'(A) = (1 - n\alpha)p\alpha\left[A^{\alpha-1}x(A)^\alpha + A^\alpha x(A)^{\alpha-1}x'(A)\right] \tag{A4}$$

From (A4) and (A2) we get

$$x'(A) = \frac{r}{b(1-n\alpha)} - \frac{x}{A} \tag{A6}$$

The solution of (A5) is

$$x(A) = \frac{rA}{2b(1-n\alpha)} + \frac{C}{A} \tag{A6}$$

where C is the constant of integration.

The integral curve for (A6) is depicted in Figure 1. If $C = 0$, we have the linear strategy described in part (i) of proposition 7. If $C > 0$, then $x(A)$ is a non-monotone function of A and as A tends to zero, x tends to infinity. This implies that A becomes zero in finite time. There are infinitely many such equilibria. Finally, if $C < 0$, then x will become zero when A is still positive, and such a strategy cannot be optimal. Equilibrium therefore requires that $C \geq 0$.

References

[1] Benchekroun, Hassan, and Ngo Van Long, 1998, Efficiency-inducing taxation for polluting oligopolists, *Journal of Public Economics 70,* 325-342.

[2] Clemhout, Simone, and Henry Wan, Jr., 1994, Differential games: economic applications, in R. J. Aumann and S. Hart (eds.) *Handbook of Game Thery, Vol 2,* Elsevier, Amsterdam.

[3] Dawkins, R, and J. Krebs, 1979, Arms races between and within species, *Proceedings of the Royal Society of London, Series B*, 489-511.

[4] Desowitz, R. 1991, *The Malaria Capers*, W.W. Norton, New York.

[5] Dockner, Engelbert, Steffen Jorgensen, Ngo Van Long, and Gerhard Sorger, 2000, *Differential Games in Economics and Management Science*, Cambridge University Press, Cambridge, UK.

[6] Feder, G. and U. Regev, 1975, Biological interactions and environmental effects in the economics of pest control, *Journal of Environmental Economics and Management 2*, 75-91.

[7] Fudenberg, D. and Jean Tirole, 1991, *Game Theory,* MIT Press, Cambridge, Mass.

[8] Hueth, D. and U. Regev, 1974, Otimal agricultural pest management with increasing pest resistance, *American Journal of Agricultural Economics 58*, 543-550

[9] Kemp, M. C. and Ngo Van Long, 1980, *Exhaustible Resources, Optimality, and Trade*, North Holland, Amsterdam.

[10] Knight, A. L. and G. W. Norton, 1989, Economics of agricultural pesticide resistance in anthropods, *Annual Review of Entomology 34*, 293-313

[11] Kogan, M, 1986, (ed.) *Ecological Theory and Integrated Pest Management Practice*, Wiley, New York.

[12] Leonard, Daniel and Ngo Van Long, 1992, *Optimal Control Theory and Static Optimization in Economics,* Cambridge University Press, Cambridge, U.K.

[13] Levy, S. B., 1992, *The Antibiotic Paradox: How Miracle Drugs are Destroying the Miracle,* Plenum Press, New York.

[14] Martin, H. and D. Woodcock, 1983, *The Scientific Principles of Crop Protection,* 7th edition, Edward Arnold, London.

[15] Metcalfe, R. L., 1955, Physiological basis for insect resistance to insecticides, *Physiological Review* 35, 192-232.

[16] Neu, H. C., 1992, The crisis in antibiotic resistance, *Science 257*, 1064-73.

[17] Regev, U., A. P. Gutierez, and G. Feder, 1976, Pests as a common property resource: a case study of alfafa weevil control, *American Journal of Agricultural Economics 60*, 168-197.

[18] Regev, U., H. Shalit and A. P. Gutierez, 1983, The optimal allocation of pesticides with increasing resistance: the case of alfafa weevil, *Journal of Environmental Economics and Management 19*, 86-100.

[19] van Valen, L. 1973, A new evolutionary law, *Evolution Theory 1*, 1-30.

[20] Wernsdorfer, W. H. and I. McGregor, 1988, *Malaria: Principles and Practice of Malariology,* Vol. 1 and 2, Churchill Livingstone, New York.

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