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A Mixed Bentham-Rawls Criterion for Intergenerational Equity

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A Mixed Bentham-Rawls Criterion for Intergenerational Equity

Ngo Van Long[*]

Résumé / *Abstract*

On propose un nouveau critère d'évaluation du bien-être social qui satisfait trois propriétés : sensibilité au bien-être des membres infortunés de la société, sensibilité au bien-être des générations futures, et sensibilité au bien-être des générations présentes. On obtient les conditions nécessaires pour le sentier optimal sous ce nouveau critère et on montre que le sentier optimal existe dans un modèle d'accumulation du capital sous des conditions normales. Le sentier optimal converge à un état stationnaire qui dépend de la condition initiale. Le long de ce sentier, la contrainte sur le niveau de bien-être minimal, qui est choisi endogènement, est satisfaite avec égalité pendant une certaine phase. Les sentiers optimaux ont des propriétés qui semblent satisfaisantes sur le plan éthique.

Mots clés : bien-être social, juste distribution, développement soutenable, équité entre les générations

This paper proposes a new welfare criterion which satisfies three desiderata: strong sensitivity to the least advantaged, sensitivity to the present, and sensitivity to the future. We develop necessary conditions for optimal paths under this new criterion, and demonstrate that, in a familiar dynamic model of capital accumulation, the optimal growth path exists. The optimal path converges to a steady state which is dependent on the initial stock of capital. Along this path, the minimum standard of living constraint, which is optimally chosen, is binding over some time interval. Optimal paths under the new criterion display properties that seem to be ethically appealing.

Keywords: *welfare, distributive justice, sustainable development, intergenerational equity*

Codes JEL : H4, I3, O2, Q56

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1 Introduction

Comparing utility streams in the infinite-time-horizon context has been a perplexing issue confronting philosophers and economists. In an article titled "A Neglected Family of Aggregation Problems in Ethics", published in Noûs (1976), the philosopher Krister Segerberg poses the following problem in ethics:

Suppose Pascal is interested in what will happen to him after his death. "He believes that eternity consists of infinitely many days [and] that when his body is dead his soul will spend each following day in Heaven or Hell...Outcomes can be represented by infinite sequences x_0 x_1... x_n..., where each x_n is either 1(Heaven) or 0(Hell)....Problems arise when he wants to compare prospects containing both 1's and 0's. Particularly difficult is it to deal with prospects containing infinitely many 1's and also infinitely many 0's."(p. 226). (He did not refer to any related work by economists.)

In economics, this type of problem is often addressed in a context that involves the utilities not of the same person in successive periods, but rather of distinct individuals in successive generations. Perhaps Ramsey (1928) was the first economist to have articulated this problem. According to Ramsey, it is unethical to discount the utilities of future generations. Various utilitarian welfare criteria that do not use discounting have been proposed. (See Diamond (1965), Koopmans (1965), von Weizsäcker (1965), Gale (1967), among others.) There are also non-utilitarian criteria such as maximin[1], and sufficientarianism.[2]

All the existing criteria have been subjected to criticism. Chichilnisky (1996) points out that the utilitarian criterion with positive discounting implies "dictatorship of the present", while criteria such the long-run average

[1] Maximin is often attributed to Rawls, but, as I have argued elsewhere (Long, 2005), in the context of intergeneraltional equity such attribution is completely unfair to Rawls.

[2] See for example Chichilnisky (1977), Frankfurt (1988), Waltzer (1983), Anderson (1999), Arneson (2002), and Roemer (2003).

criterion and the catching-up criterion imply “dictatorship of the future.” She proposed a criterion that has some desirable properties, including “non-dictatorship of the present” and “non-dictatorship of the future”[3]. The 1996-Chichilnisky welfare function is a weighted average of two terms. The first term is the usual sum of discounted utilities, and the second term depends only on the limiting properties of the utility sequence. Unfortunately, when one tries to find paths in familiar models using Chichilnisky criterion, typically one discovers that they do not exist.

In this paper, I introduce the concept of “non-dictatorship of the least advantaged” and propose a social welfare function that satisfies this property and yet embodies the Rawlsian insistence that the least advantaged deserve special considerations. This welfare function is a weighted average of (i) the usual sum of discounted utilities, and (ii) the utility level of the least advantaged generation. I call this new criterion the Mixed Bentham-Rawls criterion (MBRC). I develop a set of necessary conditions to characterize growth paths that satisfy MBRC, and show that in some models with familiar dynamic specifications, an optimal path under MBRC exists and displays appealing characteristics.

2 A theoretical framework

In order to facilitate comparison, I shall adopt a common theoretical framework in which the welfare criteria that I discuss below can be explained. I consider an economy with infinitely many generations. Since I wish to focus on the question of “distributive justice among generations”, I shall make the simplifying assumption that within each generation, all individuals receive the same income and have the same tastes. Thus, by assumption, the question of

[3]In addition to these properties, she requires that the social welfare function be Paretian and satisfy the axiom of “continuity” and “independence” (in the sense of linearity.) She showed that none of the criteria I mentioned in the preceding paragraph satisfies all the required properties.

equity within each generation does not arise. This framework has been used in, for intances, Solow (1974), Hartwick (1977), Dasgupta and Heal (1979, Chapters 9-10), Dixit et al. (1980), Long and Yang (1998), Mitra (1983), Chichilnisky (1996), and Long, Mitra and Sorger (1996).

Let c_t denote the vector of consumption (of various goods and services) allocated to the representative individual of generation t. Let $u_t \equiv u(c_t)$ be the life-time utility of this individual. (u_t is a real number, and $u(.)$ is a real-valued function). For most of what follows, I shall interpret "utility" as "standard of living"of individuals, rather than some kind of happiness they get when consuming and/or contemplating their childrens' and grandchildrens' life prospects. To fix ideas, it is convenient to assume that each individual lives for just one period. Consider for the moment two alternative projects, denoted by 1 and 2. Project i (where $i = 1, 2$) yields an infinite stream of utilities denoted by

$$\{u_t^i\}_{t=1,2,...,} \equiv \{u_1^i, u_2^i, ..., u_t^i, u_{t+1}^i, ..., ..., ...\}$$

where u_t^i stands for $u_t(c_t^i)$.

I assume that while an individual of generation t might care about the consumption vector of his/her son or daughter, c_{t+1}, and that of his/her[4] grandson or granddaughter, c_{t+2}, these vectors have no impact on the "utility" level u_t. Thus it might be preferable to refer to u_t as the "standard of living" rather than "utility" of generation t.

For simplicity of notation, I shall use the symbol $\mathbf{u}^i$ to denote the utility stream $\{u_t^i\}_{t=1,2,...,}$ .Roughly speaking, a welfare criterion is a way of ranking all possible utility streams. Let S be the set of all possible utility streams. A welfare function, denoted by W,with supercsripts to distinguish among different types, is a function that maps elements of S to the real number line. I also refer to $W(.)$ as the "social welfare function"[5].

[4] To avoid repetitive uses of his/her etc., in all that follows, when referring to hypothetical persons, I shall use the masculin gender, on the understanding that it embraces the feminin gender.

[5] This usage is quite common, see, for example, Chichilnisky (1996, p. 240), Basu

To simplify matters, I shall assume that the function $u(.)$ is bounded.

Assumption 1: (Boundedness) *Utility is bounded*

$$A \leq u(c) \leq B$$

Remark 1: *The number B is the highest possible level of utility. I shall refer to B the "Bliss Utility Level".*

In what follows, I consider only welfare functions $W(.)$ that are non-decreasing[6] in u_t. That is, if the utility level of one generation increases, the social welfare cannot decrease. This is the well known Paretian property[7].

Property P:(Paretian Property) *Welfare is non-decreasing in u_t.*

In surveying some existing welfare criteria that have been considered by economists, I shall classify welfare criteria into two classes: the class of utilitarian criteria, and the class of non-utilitarian criteria.

Utilitarian criteria permit comparing (and trading-off) an increment in the utility level of an individual (or group of individuals) with a 'decrement' (negative change) in the utility level of another individual (or group). A familiar example is the "utilitarian criterion with discounting".

Under the "Utilitarian Criterion with Discounting" (at a constant rate $\delta > 0$), social welfare is denoted by W^{δ} and is defined as follows:

$$W^{\delta}(\mathbf{u}^i) = \frac{u_1^i}{1+\delta} + \frac{u_2^i}{(1+\delta)^2} + \frac{u_3^i}{(1+\delta)^3} + ... + ...$$

and Mitra (2003). This is to be distinguished from Arrow's use of the term "social welfare function" which is a mapping from the space of all possible individual preference orderings (of social states) to the space of social orderings.

[6]We do not address the question of existence of a social welfare function $W(.)$ here. Diamond (1965) shows that if one requires that $W(.)$ must satisfy the strict Paretian property, a weak form of anonymity and some kind of continuity, then $W(.)$ does not exist. Basu and Mitra (2003) confirm Diamond's result even without requiring continuity. Svensson (1980) however shows that if, instead of seeking a (real-valued) function, we merely look for the ability to rank infinite streams of utilities, then existence (of a social welfare relation, or ordering) is ensured.

[7]The Paretian Property can be strengthened to the "Strict Paretian Property" by replacing the word "non-decreasing" by "increasing".

According to this criterion, a utility stream $\mathbf{u}^j$ is ranked higher than a utility stream $\mathbf{u}^i$ if and only if $W^\delta(\mathbf{u}^j) > W^\delta(\mathbf{u}^i)$. Thus, a small decrease in the utility level of an individual (no matter how badly off he already is) can be justified by some increase in the utility level of some other individuals.

Non-utilitarian criteria do not permit such trading off. An example of non-utilitarian criteria is the "Maximin Criterion," denoted by W^m. According to this criterion, a utility stream $\mathbf{u}^i$ is ranked higher than utility stream $\mathbf{u}^j$ if and only if the utility level of the worst off generation in stream $\mathbf{u}^i$ is higher than the utility level of the worst off generation in stream $\mathbf{u}^j$, that is, if and only if,

$$\inf \left\{u_t^i\right\}_{t=1,2,...,} > \inf \left\{u_t^j\right\}_{t=1,2,...,}$$

While many people refer to the "Maximin Criterion" as the "Rawlsian Criterion", named after John Rawls, author the influential work "A Theory of Justice"(1971, 1999), it has been argued (see, for example, Long, 2005, and references cited therein) that, in the context of welfare comparison of infinite utility streams, it is unfair to Rawls to attribute the Maximin Criterion to him. He has always insisted that such a criterion is not acceptable as a criterion for justice among generations.

3 A Review of Some Welfare Criteria

3.1 The Utilitarian Criterion With Discounting

This criterion is most widely used by economists. Any standard graduate macroeconomic textbook has at least a chapter on how a "representative, infinitely-lived individual" chooses his consumption path to maximize the present value of the stream of utility:

$$\max_{c_t} \sum_{t=1}^{\infty} \beta^t u(c_t) \text{ where } \beta = 1/(1+\delta) < 1$$

subject to an intertemporal budget constraint. In such textbooks, welfare implications of government policies are also evaluated using the same criterion.

Perhaps the main reason for the popularity of this criterion is that it gives rise to an optimization problem that can be solved in a relatively simple way. However, simplicity or solvability are not good reasons for accepting a welfare criterion. There are valid criticisms of this criterion, the most important one being its insensitivity to the utility of very distant generations.

The utilitarian criterion with discounting has been attacked by many economists, from Ramsey (1928) to Chichilnisky (1996), and others. Let me quote a forceful example from Chichilnisky (1996, page 235):

"...Discounting future utility is generally inconsistent with sustainable development. It can produce outcomes which seem patently unjust to later generations. Indeed, under any positive discount rate, the long-run future is deemed irrelevant. For example, at a standard 5% discount rate, the present value of the earth's aggregate output discounted 200 years from now, is a few hundred thousand dollars. A simple computation shows that if one tried to decide how much it is worth investing in preventing the destruction of the earth 200 years from now, the answer would be no more than one is willing to invest in an apartment."

Chichilnisky (1996) argues that all utilitarian criterion with discounting put too much emphasis on the present. In fact this criterion displays insensitivity to the utility of distant generations. To formalize this idea, let us define $({}_T\mathbf{s}^i, \mathbf{a}_T)$ to be a utility sequence obtained from $\mathbf{s}^i$ by replacing all elements of s^i except the first T elements by the tail of the utility sequence $\mathbf{a}$, where

$$\mathbf{a}_T \equiv \{a_{T+1}, a_{T+2}, ...\}$$

$${}_T\mathbf{s}^i \equiv \left\{s_1^i, s_2^i, ..., s_T^i\right\}$$

Consider the following definition:

Definition 1: (dictatorship of the present- Chichilnisky 1996)

A welfare criterion $W(.)$ is said to display "dictatorship of the present" if the following condition holds:

For every pair $(\mathbf{s}^i, \mathbf{s}^j)$*,* $W(\mathbf{s}^i) > W(\mathbf{s}^j)$ *if and only if, for all* T *sufficiently large*[8]*,* $W({}_T\mathbf{s}^i, \mathbf{a}_T) > W({}_T\mathbf{s}^j, \mathbf{b}_T)$ *for* ***all*** *pairs of sequences* $(\mathbf{a}, \mathbf{b})$*, where* $({}_T\mathbf{s}^i, \mathbf{a}_T)$ *means that all elements of* $\mathbf{s}^i$ *except the first* T *elements are replaced by the tail of the sequence* $\mathbf{a}$*, and* $({}_T\mathbf{s}^j, \mathbf{b}_T)$ *means that all elements of* $\mathbf{s}^j$ *except the first* T *elements are replaced by the tail of the sequence* $\mathbf{b}$*.*

In other words, dictatorship of the present means that any modification of utility levels of generations far away in the future would not be able to reverse the welfare ranking of two utility streams. Given our boundedness assumption, the utilitarian criterion with positive discounting displays dictatorship of the present.

A welfare function is said to display "non-dictatorship of the present" if there exists **some** pair $(\mathbf{s}^i, \mathbf{s}^j)$ such that $W(\mathbf{s}^i) > W(\mathbf{s}^j)$ and **some** modifications to utilities of individuals in the distant future can reverse the ranking.

3.2 The Long-Run Average Criterion

According to the Long-Run Average Criterion[9], stream $\mathbf{u}^i$ is declared to be better than stream $\mathbf{u}^j$ if there exists some $t_0 \geq 0$ and some number $n > 1$ such that, for all $N \geq n$ and all $t_1 \geq t_0$

$$\frac{1}{N}\left(\sum_{t=t_1}^{t_1+N} u_t^i\right) > \frac{1}{N}\left(\sum_{t=t_1}^{t_1+N} u_t^j\right)$$

The Long-Run Average Criterion favours the future generations at the expense of the present generation. Welfare comparison using this criterion depends only on the utility levels of generations born in the distant future. Chichilnisky (1996) pointed out that the Long-Run Average Criterion gives a "dictatorial role" to the future. Formally, a welfare criterion $W(.)$ is said to give a dictatorial role to the future if it has the following property:

[8] More precisely, for all $T > \widehat{T}$ for some $\widehat{T}$ that may depend on $\mathbf{u}^i$ and $\mathbf{u}^j$.

[9] There are several ways in which the Long-Run Average Criterion can be defined. However, they all have the same bias against the present generation.

Definition 2: (dictatorship of the future; Chichilnisky 1996)

A welfare criterion $W(.)$ *is said to display "dictatorship of the future" if the following condition holds:*

For every pair $(\mathbf{s}^i, \mathbf{s}^j)$, $W(\mathbf{s}^i) > W(\mathbf{s}^j)$ *if and only if, for all* T *sufficiently large*[10], $W({}_T\mathbf{a}, \mathbf{s}^i_T) > W({}_T\mathbf{b}, \mathbf{s}^j_T)$ *for* ***all*** *pairs of sequences* $(\mathbf{a}, \mathbf{b})$, *where* $({}_T\mathbf{a}, \mathbf{s}^i_T)$ *means that the first* T *elements of* $\mathbf{s}^i$ *is replaced by the vector* ${}_T\mathbf{a} \equiv (a_1, a_2, ..., a_T)$, *and* $({}_T\mathbf{b}, \mathbf{s}^j_T)$ *means that the first* T *elements of* $\mathbf{s}^j$ *is replaced by the vector* ${}_T\mathbf{b} \equiv (b_1, b_2, .., b_T)$.

A welfare function is said to display "non-dictatorship of the future" if there exists **some** pair $(\mathbf{s}^i, \mathbf{s}^j)$ such that $W(\mathbf{s}^i) > W(\mathbf{s}^j)$ and **some** modifications to utilities of individuals in the early generations can reverse the ranking.

3.3 The Distance-from-Bliss Criterion

The "Distance-from-Bliss Criterion", proposed by Ramsey (1928), is denoted by W^B. According to this criterion, stream $\mathbf{u}^i$ is ranked higher than stream $\mathbf{u}^j$ if and only if $\mathbf{u}^i$ is "closer" to the Bliss stream $B, B, B, ..., B, ...$, in the sense that

$$\sum_{t=1}^{\infty}(B - u^i_t) < \sum_{t=1}^{\infty}(B - u^j_t)$$

Note that the sums may fail to converge, in which case we must use other criteria for comparison.

3.4 The Overtaking Criterion

The Overtaking Criterion was proposed by Koopmans (1965) and von Weizsäcker (1965). According to this criterion, stream $\mathbf{u}^i$ is better than stream $\mathbf{u}^j$ if the cumulative sum (up to time T) of the differences $u^i_t - u^j_t$ is positive for all T sufficiently large.

[10] More presisely, for all $T > K$ where K may depend on $(\mathbf{s}^i, \mathbf{s}^j)$.

Formally, we define the cumulative sum (up to time T) of the differences $u_t^i - u_t^j$ as follows:

$$D_T(\mathbf{u}^i - \mathbf{u}^j) \equiv \sum_{t=1}^{T} (u_t^i - u_t^j)$$

If there exists a time $\widehat{T} > 0$ such that for all $T \geq \widehat{T}$, $D_T(\mathbf{u}^i - \mathbf{u}^j) > 0$ then under the overtaking criterion, stream $\mathbf{u}^i$ is considered to be better than $\mathbf{u}^j$. In other words, in the cumulative performance sense, stream $\mathbf{u}^i$ eventually "overtakes" stream $\mathbf{u}^j$.

3.5 The Catching-Up Criterion

A major problem with the Overtaking Criterion is that the sequence $\{D_T(\mathbf{u}^i - \mathbf{u}^j)\}_{T=1,2,3,...}$ may fail to converge to a limit. For any given $t > 0$, denote by z_t the greatest lower bound of the sequence $\{D_T(\mathbf{u}^i - \mathbf{u}^j)\}_{T=t,t+1,t+2,..,}$.

$$z_t \equiv \inf_t \left\{D_T(\mathbf{u}^i - \mathbf{u}^j)\right\}_{T=t,t+1,t+2,..,}$$

By definition $\{z_t\}_{t=1,2,3,...}$ is a monotone non-decreasing sequence, so it must have a limit in the space of extended real numbers. Thus

$$\lim_{t\to\infty} \left[\inf_t \left\{D_T(\mathbf{u}^i - \mathbf{u}^j)\right\}_{T=t,t+1,t+2,..,}\right] \text{ exists.}$$

The sequence $\mathbf{u}^i$ is said to "catch up" with the sequence $\mathbf{u}^j$ if

$$\lim_{t\to\infty} z_t \geq 0$$

The "Catching-Up Criterion", proposed by Gale (1967), says that sequence $\mathbf{u}^i$ is "no worse" than sequence $\mathbf{u}^j$ if

$$\lim_{t\to\infty} \left[\inf_t \left\{D_T(\mathbf{u}^i - \mathbf{u}^j)\right\}_{T=t,t+1,t+2,..,}\right] \geq 0$$

3.6 The Chichilnisky Non-dictatorship Criterion

Chichilnisky argued that both dictatorship of the present and dictatorship of the future are undesirable. She proposed a criterion that rules out both forms of dictatorship.

The welfare function proposed by Chichilnisky[11] takes the following form

$$W^C(\mathbf{u}^i) = \theta\phi(\mathbf{u}^i) + (1-\theta)\sum_{t=1}^{\infty}\lambda_t u_t^i$$

where $0 < \theta < 1$, $0 < \lambda_t < 1$, $\sum_{t=1}^{\infty}\lambda_t < \infty$ and

$$\phi(\mathbf{u}^i) \equiv \lim_{t\to\infty} u_t^i$$

This social welfare function clearly has the properties of "non-dictatorship of the present" and "non-dictatorship of the future".

It is interesting to observe that W^C is a weighted average (a convex combination) of two functions that are themselves based on rejected welfare criteria. The first function, $\phi(\mathbf{u}^i) = \lim_{t\to\infty} u_t^i$, implies dictatorship of the future, while the second function, $\sum_{t=1}^{\infty}\lambda_t u_t^i$, implies dictatorship of the present. A convex combination that gives strictly positive weights to two "undesirable" welfare functions is free from their associated undesirable properties.

A major problem with the Chilchinisky welfare function $W^C(.)$ is that for many growth models, including the familiar one-sector growth model, there does not exist an optimal path under this objective function. The intuition behind this non-existence is as follows. The function $\phi(\mathbf{u}^i) = \lim_{t\to\infty} u_t^i$ would insist on reaching the Golden Rule capital stock. The second function, $\sum_{t=1}^{\infty}\lambda_t u_t^i$, would insist on reaching, instead, the Modified Golden Rule capital stock. Any path $\mathbf{u}^i$ that goes near the Modified Golden Rule capital stock and eventually veers to the Golden Rule capital stock at some time T_i will

[11] This welfare function satisfies the axioms of "non-dictatorship of the present" and "non-dictatorship of the future." If two more axioms are added, "continuity" and "independence" (in the sense of linearity), then this is the only form the welfare function can take.

be beaten by another path $\mathbf{u}^j$ that does a similar thing but at a later date $T_j > T_i$. The latter path $\mathbf{u}^j$ in turn will be beaten by another path $\mathbf{u}^h$ with $T_h > T_j$ and so on. So an optimal path does not exist.

3.7 The Satisfaction of Basic Needs Criterion, or Sufficientarianism

Several economists and philosophers[12] have advocated development programs that aim at guaranteeing everybody "good-enough", or "sufficient" levels of primary goods, while allowing some people to have more than what is considered sufficient. Suppose that utility is measured such that, for some number $\widehat{u}$, if $u_t \geq \widehat{u}$ then basic needs are deemed to be satisfied at time t. Let $T(\mathbf{u}^i)$ be the time period beyond which basic needs will always be satisfied:

$$T(\mathbf{u}^i) \equiv \min \left\{ t : u^i_\tau \geq \widehat{u} \text{ for all } \tau \geq t \right\}$$

The Satisfaction of Basic Needs Criterion says that $\mathbf{u}^i$ is better than $\mathbf{u}^j$ if $T(\mathbf{u}^i) < T(\mathbf{u}^j)$. A suitable social welfare function may thus take the form:

$$W^{BN}(\mathbf{u}^i) = 1/T(\mathbf{u}^i)$$

The Satisfaction of Basic Needs Criterion (or Sufficientarianism) implies a form of insensitivity to the utility of distant future generations, i.e., it displays dictatorship of the present. Another difficulty with the notion of basic needs is the challenge of defining "good enough" levels of primary goods. Arneson (2003, p. 173) proposes that "it might be stipulated that everyone has enough income and wealth when nobody has less than some fraction of the average level." But clearly what is deemed "sufficient" in year 2000 would not be good enough for year 2050, say.

[12] See Chichilnisky (1977), Frankfurt (1988), Waltzer (1983), Anderson (1999), Arneson (2002), and Roemer (2003).

3.8 The Maximin Criterion

Rawls (1958, 1963, 1971, 1999) argues that, in contexts that *do not involve the question of justice among generations,* in deciding on whether a distribution of income is preferable to another, the only relevant things to look at are the utility levels of the worse off individual in each distribution. According to Rawls, one ought to choose the distribution with the highest utility level for the worse off individuals. This is often called the maximin criterion.[13]

Many economists have applied the maximin principle to the problem of intergenerational equity (without paying attention to Rawls' objection.) The value taken on by the social welfare function under the maximin criterion is equal to the utility level of the worse off generation:

$$W^M(\mathbf{u}^i) = \inf\left\{u_1^i, u_2^i, ..., u_n^i, ...\right\}$$

Let us define "dictatorship of the least advantaged" as follows:

Definition 3: (dictatorship of the least advantaged)[14]

A welfare criterion $W(.)$ is said to display "dictatorship of the least advantaged" if the following condition holds:

For any pair $(\mathbf{s}^i, \mathbf{s}^j)$, $W(\mathbf{s}^i) > W(\mathbf{s}^j)$ if and only if

$$\inf\left\{s_1^i, s_2^i, ..., s_n^i, ...\right\} > \inf\left\{s_1^j, s_2^j, ..., s_n^j, ...\right\}$$

As I have emphasized elsewhere (Long, 2005), in the context of intergenerational distributive justice, Rawls (1971, 1999) is opposed to the dictatorship

[13] We should note in passing that the maximin criterion does not satisfy the strict Paretian property, but a strengthened form of it, called the *leximin criterion*, does. Leximin (or lexical maximin) requires that if several streams of the utilities have the same maximin, then society must choose the one where the utility of the second worse off individual is highest, and so on. See Sen (1976), Rawls (1999, p.72), Roemer (2005), Roemer and Veneziani (2005), and Silvestre (2005).

[14] It is clear that non-dictatorship of the present and non-dictatorship of the future rule out dictatorship of the least advantaged.

of the least advantaged: ...“the difference principle does not apply to the savings problem....The principle is inapplicable and it would seem to imply...that there be no savings at all” (Rawls, 1971, p. 291). He proposed instead a principle of just savings (see Long, 2005, for a discussion; see also Appendix 2 of the present paper.)

4 A new proposal: The Mixed Bentham-Rawls Criterion

In this section, I propose a new welfare criterion which is arguably consistent with Rawls's principle of just savings. I call it the **Mixed Bentham-Rawls Criterion (or MBR criterion)**, and denote it by W^{BR} where the superscript BR refers to Bentham and Rawls. I shall at first present a basic version of the MBR criterion[15]. Subsequently, I shall discuss a more general version.

4.1 The Basic MBR Criterion

Consider a Rawlsian hypothetical original position, under the assumption that the contracting parties are family lines. A family line is at the same time “one” and “many”. Being “one”, it is like a single individual. There are no valid reasons to object to an individual's discounting of his future consumption. But a family line is also “many.” The worse off individuals have special claims not unlike the those accorded to the “contemporaneous individuals” of the simple Rawlsian model without intergenerational considerations.It is therefore arguable that each contracting party would (i) place special weight on the uitility level of the least advantaged generation, and (ii) care about the sum of weighted utilities of all generations. It seems also sensible to allow a trade-off between (i) and (ii) above, because each party represents a family line. The standard utilitarian tradition would treat a family line as an

[15] An earlier version of this new welfare criterion was stated in Long (2005).

infinitely-lived individual. This could however result in requiring great sacrifices of early generations who are typically poor. In contrast, the approach proposed here avoids imposing very high rates (of savings) at the earlier stages of accumulation.

Consider a pair of utility streams $(\mathbf{u}^i, \mathbf{u}^j)$. For a given $\theta \in [0,1]$, and any given time T, I denote by $W_T^{BR}(\mathbf{u}^i)$ the weighted average of (a) the utility of the least advantaged generation over the horizon T, and (b) the cumulative T-truncated life-time utility of the fictitious infinitely-lived individual using a discount rate $\delta \geq 0$. Thus,

$$W_T^{BR}(\mathbf{u}^i) = \theta \inf \left\{ u_t(c_t^i) : t \leq T \right\} + (1-\theta) \sum_{t=1}^{T} \beta^t u_t(c_t^i) \text{ where } \beta = \frac{1}{1+\delta} \leq 1$$

I denote by $D_T^{BR}(\mathbf{u}^i - \mathbf{u}^j)$ the difference between $W_T^{BR}(\mathbf{u}^i)$ and $W_T^{BR}(\mathbf{u}^j)$:

$$D_T^{BR}(\mathbf{u}^i - \mathbf{u}^j) \equiv W_T^{BR}(\mathbf{u}^i) - W_T^{BR}(\mathbf{u}^j)$$

The sequence $\left\{ D_T^{BR}(\mathbf{u}^i - \mathbf{u}^j) \right\}_{T=1,2,3,...}$ may fail to have a limit. Let us consider, for any given $t > 0$, the greatest lower bound of the sequence $\left\{ D_T^{BR}(\mathbf{u}^i - \mathbf{u}^j) \right\}_{T=t,t+1,t+2,..,}$. Denote this greatest lower bound by z_t^{BR}.

$$z_t^{BR} \equiv \inf_t \left\{ D_T^{BR}(\mathbf{u}^i - \mathbf{u}^j) \right\}_{T=t,t+1,t+2,..,}$$

As t gets larger and larger, z_t^{BR} can only increase or stay the same (i.e., $\left\{ z_t^{BR} \right\}_{t=1,2,3,...}$ is a monotone non-decreasing sequence), so it must have a limit in the space of extended real numbers. Thus

$$\lim_{t\to\infty} \left[\inf_t \left\{ D_T^{BR}(\mathbf{u}^i - \mathbf{u}^j) \right\}_{T=t,t+1,t+2,..,} \right] \text{ exists.}$$

If

$$\lim_{t\to\infty} z_t^{BR} \geq 0$$

then I say sequence $\mathbf{u}^i$ is able to “catch up” with sequence $\mathbf{u}^j$ under the MBR criterion.

Definition NW: A utility sequence $\mathbf{u}^i$ is said to be *no worse* than a sequence $\mathbf{u}^j$ under the Mixed Bentham-Rawls criterion if and only if

$$\lim_{t\to\infty}\left[\inf_t \left\{D_T^{BR}(\mathbf{u}^i - \mathbf{u}^j)\right\}_{T=t,t+1,t+2,..,}\right] \geq 0.$$

We denote this by $\mathbf{u}^i \succsim \mathbf{u}^j$.

Definition CU: A growth path is called "catch-up optimal under the Mixed-Bentham-Rawls Criterion" if the resulting utility sequence is no worse (in the sense of Definition NW) than any other feasible utility sequence.

Remark 1: For $\theta \in (0,1)$, the MBR criterion has the following properties:
(i) Paretian
(ii) Non-dictatorship of the future
(iii) Non-dictatorship of the present
(iv) Non-dictatorship of the least advantaged.

Clearly, the MBR welfare criterion can be seen as a compromise between the maximin criterion (which is obtained by setting $\theta = 1$) and the standard utilitarian criterion with discounting (which can be obtained by setting $\theta = 0$). However, the reason for proposing this mixed criterion is not to achieve compromise for the sake of compromising. Rather, the virtue of this new criterion is that it reflects the dual nature of a family line.

In what follows, I shall present a simple model of economic growth, and show that the use of the MBR welfare criterion does indeed generate consumption/investment paths that seem quite appealing to our notion of justice.

4.2 The Discrete-Time Maximum Principle under the MBR Criterion

I consider a dynamic system consisting of a vector of state variables x_t (interpreted as stocks of productive assets) and a vector of control variables c_t. Let

γ_t be an index of exogenous technological progress. The transition equation is

$$x_{t+1} - x_t = g(x_t, c_t, \gamma_t)$$

where $g(.)$ is twice differentiable in (x, c, γ). The period t utility function is

$$u_t = u(x_t, c_t, \gamma_t)$$

where $u(.)$ is twice differentiable in (x, c). At any time t, given x_t and γ_t, the control variables c_t must satisfy some feasibility conditions, expressed as inequality constraints

$$h(x_t, c_t, \gamma_t) \geq 0. \tag{1}$$

For any given x_1 and T, consider the T period optimization problem

$$\max W_T^m = \theta \inf_{t \leq T} [u(x_t, c_t, \gamma_t)] + (1-\theta) \sum_{t=1}^{T} \beta^t u(x_t, c_t, \gamma_t) \text{ where } \beta = \frac{1}{1+\delta} \leq 1$$

subject to the transition equation, the inequality constraints (1), and $x_{T+1} \geq 0$.

I now derive the necessary conditions for this problem. I introduce a control parameter $\underline{u}$, and require that x_t, c_t be chosen such that

$$u(x_t, c_t, \gamma_t) \geq \underline{u} \tag{2}$$

Our objective function becomes: given x_1, choose the number $\underline{u}$ and sequences $\{c_t\}_{t=1,2,...,T}$ and $\{x_t\}_{t=2,3,...,T+1}$to maximize

$$W_T^m = \theta \underline{u} + (1-\theta) \sum_{t=1}^{T} \beta^t u(x_t, c_t, \gamma_t) \tag{3}$$

subject to the constraint (2), the transition equations

$$g(x_t, c_t, \gamma_t) + x_t - x_{t+1} = 0 \text{ for } t = 1, 2, ..., T \tag{4}$$

the inequality constraints (1), and $x_{T+1} \geq 0$.

Notice that, given the technology and the initial stock x_1, if $\underline{u}$ is too great, then there is no feasible path that satisfies (2). Let $\underline{u}_{\max}$ be the least upperbound on the set of $\underline{u}$ such that feasible paths exists.

Define the over-all Lagrangian function

$$L^0 = \theta \underline{u} + (1-\theta) \sum_{t=1}^{T} \beta^t u(x_t, c_t, \gamma_t) + \sum_{t=1}^{T} \omega_t \left[u(x_t, c_t, \gamma_t) - \underline{u} \right] +$$

$$+ \sum_{t=1}^{T} \left\{ \pi_t \left[g(x_t, c_t, \gamma_t) + x_t - x_{t+1} \right] + \lambda_t h(x_t, c_t, \gamma_t) \right\}$$

The necessary conditions are

$$\frac{\partial L^0}{\partial c_t} = \left[\omega_t + (1-\theta)\beta^t \right] u_c(x_t, c_t, \gamma_t) + \pi_t g_c(x_t, c_t, \gamma_t) + \lambda_t h_c(x_t, c_t, \gamma_t) = 0 \text{ for } t = 1, 2, ..., T$$

where

$$\lambda_t \geq 0 \ , \ h(x_t, c_t, \gamma_t) \geq 0 \text{ and } \lambda_t h(x_t, c_t, \gamma_t) = 0$$

$$\frac{\partial L^0}{\partial x_t} = \left[\omega_t + (1-\theta)\beta^t \right] u_x(x_t, c_t, \gamma_t) - \pi_{t-1} + \pi_t \left[1 + g_x(x_t, c_t, \gamma_t) \right] + \lambda_t h_x(x_t, c_t, \gamma_t) = 0 \text{ for } t = 2, 3, ...,$$

$$\frac{\partial L^0}{\partial x_{T+1}} = -\pi_T \leq 0 \text{ (with } \pi_T x_{T+1} = 0)$$

$$\frac{\partial L^0}{\partial \pi_t} = g(x_t, c_t, \gamma_t) + x_t - x_{t+1} = 0 \text{ for } t = 1, 2, ..., T$$

$$\frac{\partial L^0}{\partial \underline{u}} = \theta - \sum_{t=1}^{T} \omega_t \geq 0 \ (= 0 \text{ if } \underline{u} < \underline{u}_{\max})$$

$$\omega_t \geq 0, \ u(x_t, c_t, \gamma_t) - \underline{u} \geq 0 \text{ and } \omega_t \left[u(x_t, c_t, \gamma_t) - \underline{u} \right] = 0$$

It will be useful to rearrange the above necessary conditions in terms of the following present-value Hamiltonian function and present-value Lagrangian function

$$H(t, x_t, c_t, \pi_t) \equiv (1-\theta)\beta^t u(x_t, c_t, \gamma_t) + \pi_t g(x_t, c_t, \gamma_t)$$

$$L(t, x_t, c_t, \pi_t, \lambda_t, \omega_t) \equiv H(t, x_t, c_t, \pi_t) + \lambda_t h(x_t, c_t, \gamma_t) + \omega_t \left[u(x_t, c_t, \gamma_t) - \underline{u}\right]$$

Then the necessary conditions can also be stated as follows.

The discrete-time maximum principle with optimal self-imposed minimum standard of living:

(i) **The maximum condition:** The vector of control variables c_t maximizes the Hamiltonian $H(t, x_t, c_t, \pi_t)$ subject to the inequality constraints (1) and (2):

$$\frac{\partial L(t, x_t, c_t, \pi_t, \lambda_t, \omega_t)}{\partial c_t} = \left[\omega_t + (1-\theta)\beta^t\right] u_c(x_t, c_t, \gamma_t) + \pi_t g_c(x_t, c_t, \gamma_t) + \lambda_t h_c(x_t, c_t, \gamma_t) = 0$$

for $t = 1, 2, ..., T$,where

$$\lambda_t \geq 0 \text{ , } h(x_t, c_t, \gamma_t) \geq 0 \text{ and } \lambda_t h(x_t, c_t, \gamma_t) = 0$$

(ii) **The adjoint equations:** The evolution of the vector of shadow prices π_t satisfies the "arbitrage condition"

$$\pi_t - \pi_{t-1} = -\frac{\partial L(t, x_t, c_t, \pi_t, \lambda_t, \omega_t)}{\partial x_t} = -\left[\omega_t + (1-\theta)\beta^t\right] u_x(x_t, c_t, \gamma_t) - \lambda_t h_x(x_t, c_t, \gamma_t)$$

for $t = 1, 2, ..., T$,

(iii) **The transition equations** satisfy

$$x_{t+1} - x_t = \frac{\partial L(t, x_t, c_t, \pi_t, \lambda_t, \omega_t)}{\partial \pi_t} = g(x_t, c_t, \gamma_t)$$

for $t = 2, 3, ..., T+1$.

(iv) **The optimal control parameter** $\underline{u}$ satisfies

$$\theta - \sum_{t=1}^{T} \omega_t \geq 0 \text{ (} = 0 \text{ if } \underline{u} < \underline{u}_{\max})$$

$$\omega_t \geq 0 \text{ and } \omega_t \left[u(x_t, c_t, \gamma_t) - \underline{u}\right] = 0$$

(v) **The transversality condition** on terminal stock is

$$x_{T+1} \geq 0, \pi_T \geq 0, \, \pi_T x_{T+1} = 0.$$

Remark 2: The necessary conditions can be extended to the infinite horizon case.

5 Application to a growth model in discrete time

Consider an economy endowed at time $t = 1$ with a stock of renewable resource, denoted by x_1. Let $c_t \leq x_t$ be the amount harvested in period t for consumption by generation t. We assume that the transition equation is

$$x_{t+1} - x_t = (x_t - c_t)^{\alpha} \quad - x_t \equiv g(x_t, c_t) \text{ where } 0 < \alpha < 1$$

The utility of generation t is $u(c_t) = \ln c_t$. The initial stock x_1 is given. Under our proposed welfare criterion, given any T , the planner finds the time path of consumption to maximize welfare under the mixed Bentham-Rawls criterion.

$$W = \theta \inf [\ln c_t] + (1 - \theta) \sum_{t=1}^{\infty} \beta^t \ln c_t \text{ where } \beta = \frac{1}{1 + \delta} < 1$$

subject to the transition equation and $x(t) \geq 0$. Before solving this problem, let us look at two polar cases. In the first polar case, $\theta = 0$, so that the objective function reduces to the conventional utilitarian criterion. In the second polar case, $\theta = 1$, so the welfare criterion is the maximin one.

5.1 The polar utilitarian case $\theta = 0$

To find the solution for this polar case, we consider the Bellman equation

$$V(x_t) = \max_{c_t} [\ln c_t + \beta V((x_t - c_t)^{\alpha})]$$

where $V(x_t)$ is the value function, and $\beta = 1/(1 + \delta)$. We obtain the Euler equation

$$\frac{u'(c_{t+1})}{u'(c_t)} = \frac{1 + \delta}{\alpha(x_t - c_t)^{\alpha - 1}} \equiv \frac{1 + \delta}{1 + r_t}$$

where $1 + r_t$ is the gross marginal product of the stock x_t. This equation has the usual implication: if the rate of interest r_t exceeds the rate of utility discount δ, then consumption should rise, i.e., $c_{t+1} > c_t$. Using the fact that $u'(c_t) = 1/c_t$, we get the explicit Euler equation

$$\frac{c_t}{c_{t+1}} = \frac{1+\delta}{\alpha(x_t - c_t)^{\alpha-1}}$$

It is easy to show that following policy rule is optimal

$$c_t = (1 - \alpha\beta)x_t$$

The steady state stock is

$$x_{ss} = (\alpha\beta)^{\alpha/(1-\alpha)}$$

To illustrate, assume $x_1 = 0.2, \alpha = 0.5$ and $\beta = 0.8$. Then $x_{ss} = 0.40$ and $c_{ss} = 0.24$.

The optimal utilitarian path (with $\theta = 0$) is given below. Convergence is very fast.

period	x_t	c_t
1	0.2	0.12
2	0.28284	0.1697
3	0.33636	0.20182
4	0.3668	0.22008
5	0.38304	0.22982
6	0.39143	0.23468

5.2 The polar maximin case $\theta = 1$

Turning to the other polar case where $\theta = 1$, it is optimal to set c_t equal to the amount c_R that would maintain a constant stream of consumption (and the stock is constant over time). Thus c_R is the solution of

$$x_2 = (x_1 - c_R)^\alpha$$

with $x_2 = x_1$. Solving for the case $x_1 = 0.2$:

$$c_R = x_1 - x_1^2 = 0.2 - 0.04 = 0.16$$

which is higher than the consumption level c_1 obtained under the pure utilitarian case ($\theta = 0$). The stock remains constant $x_t = x_1 = 0.2$ for all $t \geq 1$.

5.3 Optimal path under the MBR Criterion

Assume $\theta = 0.65$. Then, given $x_1 = 0.2$, the optimal paths of stock and consumption are given below. The minimum standard of living constraint is binding for two periods. From period 3 onward, the economy behaves as if it were operating under the pure utilitarian regime from that date. Figure 1 shows the optimal path.

period	x_t	c_t
1	0.2	0.15
2	0.22361	0.15
3	0.27131	0.16279
4	0.32943	0.19766
5	0.363	0.2178
6	0.38105	0.228 63

6 The Continuous-Time Maximum Principle under the MBR Criterion

Let $\beta(t) \in (0, 1]$ be a discount factor, $-\dot{\beta}/\beta \equiv \rho(t)$ the discount rate, and $\gamma(t)$ an index of exogenous technological progress. Consider first the case of a finite horizon T. Given x_1, we seek to maximize

$$\theta \underline{u} + \int_0^T \beta(t)(1-\theta)u(x(t), c(t), \gamma(t))dt$$

subject to

$$u(x(t), c(t), \gamma(t)) \geq \underline{u} \tag{5}$$

$$h(x(t), c(t), \gamma(t)) \geq 0 \tag{6}$$

$$\dot{x}(t) = g(x(t), c(t), \gamma(t))$$

and

$$x(T) \geq 0$$

We assume that the technological progress index $\gamma(t)$ is a differentiable function of time. Note that $\underline{u}$ must belong to a feasible set $Z(x_0)$. In particular, we require $\underline{u} \leq \underline{u}_{\max}$. (Clearly $\underline{u}_{\max}$ depends on the initial stock.)

6.1 The Necessary Conditions

The Hamiltonian for this problem is

$$H(t,x(t),c(t),\pi(t)) \equiv (1-\theta)\beta(t)u(x(t),c(t),\gamma(t)) + \pi(t)g(x(t),c(t),\gamma(t))$$

and the Lagrangian is

$$L(t,x(t),c(t),\pi(t),\lambda(t),\omega(t),\underline{u}) = H + \lambda(t)h(x(t),c(t),\gamma(t))$$
$$+\omega(t)\left[u(x(t),c(t),\gamma(t)) - \underline{u}\right]$$

The necessary conditions may be stated as follows:

The maximum principle with control parameters (Hestenes Theorem)[16]

(i) **The maximum condition:** The control variables maximize the Hamiltonian subject to inequality constraints (5) and (6)

(ii) **The adjoint equations**:

$$\dot{\pi} = -\frac{\partial L}{\partial x}$$

(iii) **The transition equations:**

$$\dot{x} = \frac{\partial L}{\partial \pi}$$

(iv) **The Hestenes transversality conditions:**

$$\theta + \int_0^T \frac{\partial L}{\partial \underline{u}} dt \geq 0 \ (= 0 \text{ if } \underline{u} < \underline{u}_{\max})$$

and

$$x(T) \geq 0,\ \pi(T) \geq 0,\ \pi(T)x(T) = 0$$

(v) The Hamiltonian and the Lagrangian are continuous functions of time, and

$$\frac{d}{dt}H(t,x(t),c(t),\pi(t)) = \frac{d}{dt}L(t,x(t),c(t),\pi(t),\lambda(t),\omega(t),\underline{u}) = \frac{\partial L}{\partial t}$$

[16] See Leonard and Long (1991), Theorem 7.11.1 for an exposition of Hestenes' Theorem.

6.2 Implications for Genuine Savings

Let us define "present-value genuine savings" by

$$S(t) \equiv \pi(t)g(x(t), c(t), \gamma(t))$$

and "current-value genuine savings" by

$$S^c(t) \equiv \frac{\pi(t)}{\beta(t)} g(x(t), c(t), \gamma(t))$$

Then, by definition of H and S,

$$\frac{d}{dt}H = (1-\theta)\dot{\beta}u(t) + (1-\theta)\beta\dot{u} + \dot{S} \tag{7}$$

On the other hand,

$$\frac{\partial L}{\partial t} = (1-\theta)\dot{\beta}u(t) + \pi(t)g_\gamma(.)\dot{\gamma} + \lambda(t)h_\gamma(.)\dot{\gamma} + [(1-\theta)\beta + \omega]\, u_\gamma\dot{\gamma} \tag{8}$$

Using (v), it follows that along the optimal path, utility is rising at time t if and only if the *rate of change* in present-value genuine savings, adjusted for technological progress impact (the term inside the curly brackets in the equation below), is negative:

$$\dot{S} - \{\pi g_\gamma + \lambda h_\gamma + [(1-\theta)\beta + \omega]\, u_\gamma\}\,\dot{\gamma} = -(1-\theta)\beta\dot{u} \tag{9}$$

i.e.

$$\dot{S} + (1-\theta)\beta\dot{u} = \{\pi g_\gamma + \lambda h_\gamma + [(1-\theta)\beta + \omega]\, u_\gamma\}\,\dot{\gamma} \tag{10}$$

Thus,the constancy of present-value genuine savings ($\dot{S} = 0$) is consistent with growing utility if technological progress impact is positive. In particular, suppose the technological progress impact is zero. Then, as is clear from (9), if utility reaches a peak at some time $\widehat{t}$, present-value genuine savings must be at its local minimum at $\widehat{t}$. (That is, S is a mirror image of u).

Now, by definition,

$$S(t) = \beta(t)S^c(t)$$

So

$$\dot{S} = \dot{\beta} S^c + \beta \dot{S}^c$$

Assume $\dot{\beta} < 0$, e.g. $-\dot{\beta}/\beta \equiv \rho(t) > 0$. Then $\dot{S}(\widehat{t}) = 0$ is consistent with $S^c(\widehat{t}) < 0$ and $\dot{S}^c(\widehat{t}) < 0$, with

$$\frac{\dot{S}^c(\widehat{t})}{S^c(\widehat{t})} = -\frac{\dot{\beta}(\widehat{t})}{\beta(\widehat{t})} = \rho(\widehat{t}). \tag{11}$$

Proposition 1:

(a) Over any time interval where utility is constant, it holds that the rate of change in present-value genuine savings is zero (if there is no technological progress.)

(b) Zero growth in genuine savings ($\dot{S} = 0$) is consistent with positive growth in utility only if technological progress is positive.

(c) Assume there is no exogenous technological progress. If utility reaches a peak at some time $\widehat{t}$, then "genuine savings" is at a local minimum at $\widehat{t}$, and "current-value genuine savings" may have reached a peak before time $\widehat{t}$.

Proof: Parts (a) and (b) follows from (10). Part (c) follows from (10) and (11). If utility reaches a peak at some time $\widehat{t}$,then $\dot{u}(\widehat{t}) = 0$ and $\ddot{u}(\widehat{t}) < 0$, implying $\dot{S}(\widehat{t}) = 0$ and $S''(\widehat{t}) > 0$.

Remark 3: Proposition 1 is a generalization of the first proposition of Hamilton and Withagen (2006), and Hamilton and Hartwick (2005).

6.3 Infinite horizon optimization under the MBR criterion

Suppose the time horizon is infinite and the rate of discount ρ is a positive constant. Then the objective function becomes

$$\max \int_0^\infty \theta \underline{u} \rho e^{-\rho t} dt + \int_0^\infty (1-\theta) u(x,c,\gamma) e^{-\rho t} dt$$

Let $\psi(t) = \beta(t)^{-1}\pi(t)$, $\mu(t) = \beta(t)^{-1}\lambda(t)$ and $w(t) = \beta(t)^{-1}\omega(t)$. The current value Hamiltonian is

$$H = \theta \underline{u} \rho + (1-\theta) u(x,c,\gamma) + \psi g(x,c,\gamma)$$

and the current-value Lagrangian is

$$L = H + \mu h(x, c, \gamma) + w\left[u(x, c, \gamma) - \underline{u}\right]$$

Then

$$\frac{\partial L}{\partial c} = (1-\theta)u_c + \psi g_c + \mu h_c = 0$$

$$\dot{\psi} = \rho\psi - \frac{\partial L}{\partial x}$$

$$\dot{x} = \frac{\partial L}{\partial \psi}$$

The optimality condition with respect to the control parameter is

$$\int_0^\infty e^{-\rho t}\frac{\partial L}{\partial \underline{u}}dt \geq 0 \ (= 0 \text{ if } \underline{u} < \underline{u}_{\max})$$

And the following transversality condition is part of the sufficient conditions

$$\lim_{t\to\infty} e^{-\rho t}\psi(t) \geq 0, \text{ and } \lim_{t\to\infty} e^{-\rho t}\psi(t)x(t) = 0.$$

7 Optimal Renewable Resource Use under the MBR Criterion

We consider a model of optimal management of a renewable resource. The resource stock is $x(t)$. Its growth function is

$$\dot{x} = G(x) - c$$

where $G(x)$ is a strictly concave function which reaches a maximum at some $\overline{x} > 0$. We call $\overline{x}$ the "maximum sustainable yield" stock level. Assume $G(0) = 0$ and $G'(0) > 0$. The variable c is the harvest rate.

The utility function is

$$u = U(x, c)$$

which is homothetic, strictly concave and increasing, with $U_{cx} \geq 0$, $U_c(x, 0) = \infty$ and $U_x(0, c) = \infty$.

We define the "**Golden Rule stock level**" as the stock level that maximizes long-run sustainable utility:

$$\max_x U(x, G(x))$$

The golden rule stock level, denoted by x_g, is uniquely determined by the equation

$$\frac{U_x(x_g, G(x_g))}{U_c(x_g, G(x_g))} = -G'(x_g)$$

Clearly, $x_g > \overline{x}$.

By the "**Modified Golden Rule stock level**", we mean the stock level that satisfied the equation

$$\frac{U_x(x, G(x))}{U_c(x, G(x))} = -G'(x) + \rho$$

This stock level is denoted by x_ρ. Clearly

$$x_\rho < x_g$$

(This is because as we move along the curve $c = G(x)$ toward greater values of x, the ratio $G(x)/x$ falls, so $U_x(x, G(x))/U_c(x, G(x))$ falls, and thus $[U_x(x, G(x))/U_c(x, G(x))] + G'(x)$ is a decreasing function of x.)

Now consider the optimal growth program under the Mixed Bentham-Rawls objective function. Assume $0 < \theta < 1$.

$$\max \theta \underline{u} \ + (1-\theta) \int_0^\infty e^{-\rho t} U(x, c) dt$$

subject to

$$\dot{x} = G(x) - c$$

$$U(x, c) \geq \underline{u}$$

where $x(0) = x_0 > 0$.

An interesting question is: under the MBR criterion, does the optimal path approach a steady state that is somewhere between x_ρ and x_g? Proposition 2 below gives the answer.

Proposition 2: Under the MBR criterion, the steady state depends on whether the initial stock x_0 is smaller or greater than x_ρ.

(i) If $x_0 > x_\rho$, the optimal path consists of two phases. Phase I begins at $t = 0$ and ends at some finite $T > 0$. During Phase I, utility level and the resource stock are both falling. Genuine saving is negative and rises toward zero. At time T, the pair (x, c) reaches a steady-state pair (x_θ, c_θ) where $c_\theta = G(x_\theta)$. In particular,

$$x_\rho < x_\theta < x_g$$

During Phase II, the system stays at the steady state (x_θ, c_θ). Genuine saving is constant and equal to zero. (See Figure 2.)

(ii) If $x_0 < x_\rho$, the optimal path consists of two phases. Phase I begins at $t = 0$ and ends at some finite $T > 0$. During Phase I, utility is constant, which implies a time path of falling harvest rate, and rising stock. Genuine saving in this phase is positive and its rate of change is zero. In Phase II, the economy follows the standard utilitarian path and approaches asymptotically the Modified Golden Rule stock level x_ρ. Genuine saving is positive and falls steadily toward zero.

Proof:

To solve the problem, we define the current-value Hamiltonian and Lagrangian:

$$H = \theta \underline{u} \rho + (1-\theta)U(x,c) + \psi\left[G(x) - c\right]$$

$$L = H + w\left[U(x,c) - \underline{u}\right]$$

The optimality conditions are

$$\frac{\partial L}{\partial c} = (1-\theta+w)U_c - \psi = 0 \tag{12}$$

$$\dot{\psi} = \psi\left[\rho - G'(x)\right] - (1-\theta+w)U_x = (1-\theta+w)U_c\left\{\rho - G'(x) - \frac{U_x}{U_c}\right\}$$

Let us define

$$p \equiv \frac{\psi}{1-\theta+w} \tag{13}$$

Then equation (12) yields the optimal control c as a function of p and x. That is,

$$U_c(x,c) = p$$

implies

$$c = c(x,p)$$

where

$$U_{cc}\frac{\partial c}{\partial p} = 1$$

$$U_{cx} + U_{cc}\frac{\partial c}{\partial x} = 0$$

$$c_p(x,p) = -\frac{1}{U_{cc}} > 0$$

$$\frac{\partial c}{\partial x} = -\frac{U_{cx}}{U_{cc}} \geq 0$$

Note that $\dot{\psi} = 0$ iff

$$\frac{U_x}{U_c} = \rho - G'(x)$$

The question is: is it optimal to approach x_ρ where $\dot{\psi} = 0$? Can $\dot{\psi} > 0$ and yet (c,x) stay constant at a steady state? The answer is that if $x_0 > x_\rho$ we should approach a steady state $x_\theta > x_\rho$ where $\dot{p} = 0$. In what follows, we construct the path that satisfies all the necessary conditions, and then apply the sufficiency theorem to show that it is the optimal path.

From the definition of p

$$\dot{p} = \frac{\dot{\psi}}{1-\theta+w} - p\left(\frac{\dot{w}}{1-\theta+w}\right)$$

Thus

$$\dot{p} = U_c\left\{\rho - G'(x) - \frac{U_x}{U_c}\right\} - p\left(\frac{\dot{w}}{1-\theta+w}\right)$$

Now we prove part (i) of the Proposition:

Suppose $x_0 > x_\rho$. It is feasible to approach the modified golden rule stock x_ρ along a path with monotone non-increasing consumption. But it is not be optimal to do so, because the worse off individuals (in the far distant generations) can be made better off by approaching $x_\theta > x_\rho$.

To see this formally, we note that if we approach x_ρ then $w(t) = 0$ for all finite t, thus violating the transversality condition that

$$\theta = \int_0^\infty e^{-\rho t} w(t) dt$$

So the optimal path must reach, in finite time, a steady state stock level x_θ where

$$x_\rho < x_\theta < x_g$$

At x_θ,

$$\frac{U_x(x_\theta, G(x_\theta))}{U_c(x_\theta, G(x_\theta))} < \frac{U_x(x_\rho, G(x_\rho))}{U_c(x_\rho, G(x_\rho))} = \rho - G'(x_\rho) < \rho - G'(x_\theta)$$

At the steady state x_θ, $\rho - G'(x) - \frac{U_x}{U_c} > 0$, but $\dot{p} = 0$ as long as $\dot{w}$ satifies the condition

$$\frac{\dot{w}}{1 - \theta + w} = \frac{U_c}{p^*} \left\{ \rho - G'(x_\theta) - \frac{U_x(x_\theta, G(x_\theta))}{U_c(x_\theta, G(x_\theta))} \right\}$$

where

$$p^* = U_c(x_\theta, G(x_\theta))$$

At the steady state x_θ

$$\frac{\dot{\psi}}{\psi} = \rho - G'(x_\theta) - \frac{U_x(x_\theta, G(x_\theta))}{U_c(x_\theta, G(x_\theta))} > 0$$

The transversality condition

$$\lim_{t \to \infty} e^{-\rho t} \psi(t) x(t) = 0$$

is satisfied because

$$\lim_{t \to \infty} e^{-\rho t} \psi(t) = \lim_{t \to \infty} e^{-\rho t} A \exp \left\{ \left[\rho - G'(x_\theta) - \frac{U_x(x_\theta, G(x_\theta))}{U_c(x_\theta, G(x_\theta))} \right] t \right\} = 0$$

since, for $x_\theta < x_g$, it holds that

$$-G'(x_\theta) < \frac{U_x(x_\theta, G(x_\theta))}{U_c(x_\theta, G(x_\theta))}$$

It is not possible to find a closed form expression for x_θ because x_θ depends on the initial stock x_0. But we can state the conditions that must be satisfied.

Starting from $x_0 > x_\rho$, there are two phases.

In Phase I, utility is strictly falling, and $u > \underline{u}$, so that $w(t) = 0$. During this phase, the harvest rate satisfies the condition

$$(1-\theta)U_c - \psi = 0$$

hence

$$c = c^*(x, \psi/(1-\theta))$$

The evolution of ψ in Phase I is described by

$$\dot{\psi} = (1-\theta+w)U_c(x, c^*(x, \psi/(1-\theta))) \left\{ \rho - G'(x) - \frac{U_x(x, c^*(x, \psi/(1-\theta)))}{U_c(x, c^*(x, \psi/(1-\theta)))} \right\}$$

In Phase II, c is a constant, $\dot{\psi} > 0$ but $\dot{p} = 0$. During this phase

$$\frac{\dot{w}}{1-\theta+w} = \rho - G'(x_\theta) - \frac{U_x(x_\theta, c_\theta)}{U_c(x_\theta, c_\theta)} \equiv q(x_\theta) < \rho \text{ since } x_\theta < x_g \qquad (14)$$

where

$$c_\theta \equiv c(x_\theta, p^*) = G(x_\theta)$$

and

$$q'(x_\theta) > 0, \lim_{x_\theta \to x_\rho} q(x_\theta) = 0, \lim_{x_\theta \to x_g} q(x_\theta) = \rho$$

Let T denote the transition time from Phase I to Phase II. The following transversality condition must be met

$$\theta = \int_T^\infty e^{-\rho t} w(t) dt \qquad (15)$$

where $w(T) = 0$. Thus, from (14), and $w(T) = 0$, we get, for $t \geq T$

$$w(t) = (1-\theta)e^{q(t-T)} - (1-\theta)$$

Substituting into (15)

$$\theta = (1-\theta)e^{-qT}\int_T^\infty e^{-(\rho-q)t}dt - (1-\theta)\int_T^\infty e^{-\rho t}dt$$

Thus

$$e^{\rho T} = \left(\frac{1-\theta}{\theta}\right)\frac{q}{\rho(\rho-q)}$$

This equation requires T to be an increasing function of q and hence an increasing function of x_θ :

$$T = \widetilde{T}(x_\theta) \tag{16}$$

Now consider Phase I. During this phase, $w(t) = 0$. We have two differential equations

$$\dot{x} = G(x) - c(x,p)$$

$$\dot{p} = U_c\left\{\rho - G'(x) - \frac{U_x}{U_c}\right\}$$

with boundary conditions, $x(0) = x_0$, $x(T) = x_\theta$ and $p(T) = U_c(x_\theta, G(x_\theta))$. These equations yield

$$T = \widehat{T}(x_0, x_\theta) \tag{17}$$

where $\frac{\partial\widehat{T}}{\partial x_0} < 0$ and $\frac{\partial\widehat{T}}{\partial x_\theta} > 0$.

The two equations (**??**) and (**??**) yield

$$\widetilde{T}(x_\theta) - \widehat{T}(x_0, x_\theta) = 0$$

from which we obtain

$$\widetilde{T}'(x_\theta)dx_\theta - \frac{\partial\widehat{T}}{\partial x_\theta}dx_\theta - \frac{\partial\widehat{T}}{\partial x_0}dx_0 = 0$$

thus

$$\frac{dx_\theta}{dx_0} = \frac{\frac{\partial\widehat{T}}{\partial x_0}}{\left[\widetilde{T}'(x_\theta) - \frac{\partial\widehat{T}}{\partial x_\theta}\right]} \tag{18}$$

Then

$$x_\theta = X(x_0).$$

It remains to show that $X(.)$ is an increasing function for all $x_0 > x_\rho$ (i.e., that the denominator of (18) is negative), and

$$\lim_{x_0 \to x_\rho} X(x_0) = x_\rho.$$

Part (ii) of the Proposition can be proved in a similar way.

8 Concluding Remarks

In this essay, I have reviewed a number of welfare criteria for comparing long-term investment projects, and proposed a new one, called the Mixed Bentham-Rawls Criterion, that I believe does justice to the Rawlsian notion of intergenerational equity. I have restricted attention to the problem of intergenerational equity, and to facilitate the analysis, I have abstracted from intra-generational equity.

I have shown that optimal growth paths under the MBR criterion can be characterised using standard techniques. These paths seem intuitively plausible, and reflects both the Rawlsian concerns for the least advantaged, and the utilitarian principle. I have also obtained a characterization of the relationship between the growth rate of genuine savings and the growth rate of utility.

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APPENDIX 1: An example in continuous time with zero discount

We consider a model of exploitation of a renewable resource. The resource stock is denoted by $x(t) \geq 0$. Let $\alpha(t) \geq 0$ be the effort level. The transition equation is

$$\dot{x}(t) = x(t) - x(t)^2 - \alpha(t)x(t) \tag{19}$$

The quantity $\alpha(t)x(t)$ is called the harvest rate. Assume that the utility derived from $\alpha(t)x(t)$ is

$$u(\alpha x) = -(\alpha x)^{-1}$$

The maximum sustainable yield is denoted by y^M, and is defined by

$$y^M = \max_x (x - x^2) = \frac{1}{4}$$

The maximum sustainable utility is

$$u^M = -(y^M)^{-1} = -4$$

The objective function is to maximize

$$\theta \inf_t u(\alpha(t)x(t)) + (1-\theta)\int_0^\infty \left\{u(\alpha(t)x(t)) - u^M\right\} dt$$

subject to the transition equation, the constraint $x(t) \geq 0$ and the initial condition $x(0) = x_0$.

We introduce the control parameter $\underline{u}$ and seek

$$W \equiv \max \theta \underline{u} + (1-\theta)\int_0^\infty \left\{u(\alpha(t)x(t)) - u^M\right\} dt$$

$$W = \max \theta \underline{u} + (1-\theta)\int_0^\infty \left\{-(\alpha(t)x(t))^{-1} - u^M\right\} dt$$

subject to $u(\alpha(t)x(t)) \geq \underline{u}$, and (19).

The solution can be made simple by the following transformation of variable

$$s \equiv x^{-1}$$

Then

$$\dot{s} = -x^{-2}\dot{x} = -x^{-2}\left[(1-\alpha)x - x^2\right] = 1 - (1-\alpha)s$$

Then the objective function becomes

$$\max \theta \underline{u} + (1-\theta)\int_0^\infty \left\{-s(t)(\alpha(t))^{-1} + 4\right\} dt$$

subject to

$$-s\alpha^{-1} \geq \underline{u}$$

$$\dot{s} = 1 - (1-\alpha)s$$

$$s(t) \geq 0 \text{ and } \alpha(t) \geq 0.$$

Note that when $s = 0$, we have $\dot{s} = 1 > 0$, so $s(t)$ is always positive, given that $s(0) > 0$.

The Hamiltonian is

$$H = (1-\theta)\left\{-s\alpha^{-1} + 4\right\} + \pi\left[1 - (1-\alpha)s\right]$$

The Lagrangian is

$$L = H + \omega\left[-s\alpha^{-1} - \underline{u}\right]$$

The necessary conditions are

(i) $\alpha(t)$ maximizes H

$$(1-\theta+\omega)s\alpha^{-2} + \pi s = 0$$

This gives

$$\alpha = \left[\frac{1-\theta+\omega}{-\pi}\right]^{1/2} > 0 \text{ for all } \pi < 0 \tag{20}$$

Note that we can expect π to be negative, because it is the value of a marginal increase in s, which is a marginal decrease in the true stock x.

(ii) The adjoint equation is

$$\dot{\pi} = -\frac{\partial L}{\partial s} = (1-\theta+\omega)\alpha^{-1} + \pi(1-\alpha) \tag{21}$$

(iii) The Hestenes transversality condition is

$$\theta + \int_0^\infty \frac{\partial L}{\partial \underline{u}} dt \geq 0 \ (= 0 \text{ if } \underline{u} < \underline{u}_{\max})$$

i.e.

$$\theta - \int_0^\infty \omega(t) dt \geq 0$$

Substitute (20) into (21)

$$\dot{\pi} = (-\pi) \left\{ \frac{2(1-\theta+\omega)^{1/2}}{(-\pi)^{1/2}} - 1 \right\}$$

Let us consider two phases. In the first phase, when s is large (i.e., x is small), the effort level $\alpha(t)$ is chosen so that utility is constant while x is allowed to grow (s is allowed to fall). In the second phase, $\alpha(t)$ is kept constant, and utility grows.

Consider the second phase. Let us conjecture that $\dot{\pi} = 0$ in this growing utility phase. Then

$$\frac{(1-\theta+\omega)^{1/2}}{(-\pi)^{1/2}} = \frac{1}{2}$$

that is $\alpha(t) = 1/2$. During this phase

$$\dot{s} = 1 - \frac{1}{2} s$$

This differential solution has the solution

$$s(t) = 2 + Ae^{-t/2}$$

Thus $s(t)$ converges to the steady state stock level $s^* = 2$ (i.e., $x(t)$ converges to the maximum sustainable yield stock $x^* = 1/2$).

Suppose the growing utility phase starts at time T. Then

$$s(T) = 2 + Ae^{-T/2}$$

ie

$$A = [s(T) - 2] e^{T/2}$$

So

$$s(t) = 2 + [s(T) - 2]\, e^{T/2} e^{-t/2}$$

The integral of utility is

$$V_T = (1-\theta) \int_T^{\infty} \left\{ -s(t)(\alpha(t))^{-1} + 4 \right\} dt = (1-\theta) \int_T^{\infty} \{4 - 2s(t)\}\, dt$$

$$V_T = (1-\theta) \int_T^{\infty} \left\{ -2\, [s(T) - 2]\, e^{T/2} e^{-t/2} \right\} dt = -4(1-\theta)(s_T - 2)$$

Now consider the first phase. Assume $s(0) > s_T$.

At time T, the utility flow is

$$\underline{u} = u(T) = -\alpha(T)^{-1} s_T = -2s_T$$

During the first phase $[0, T]$, the utility flow is constant

$$u = \underline{u} = -2s_T$$

This means

$$-\alpha(t)^{-1} s(t) = -2s_T$$

Thus

$$\alpha(t) = \frac{s(t)}{2s_T}$$

(During the first phase, as $s(t)$ falls toward s_T, $\alpha(t)$ falls toward 1/2.)

Substituting into the transition equation:

$$\dot{s} = 1 - s + \alpha s = 1 - s + \frac{1}{2s_T} s^2$$

Given s_0 and $s_T (= -\underline{u}/2)$, we can integrate this equation to find $T = T(\underline{u})$. The integral of utility in phase I is

$$V_1 = (1-\theta) \underline{u} T(\underline{u})$$

The total welfare is

$$W = \theta\underline{u} + V_1 + V_T = \theta\underline{u} + (1-\theta)\underline{u}T(\underline{u}) - 4(1-\theta)(-\frac{1}{2}\underline{u} - 2)$$

The optimal $\underline{u}$ can then be determined. It can be verified that the optimal $\underline{u}$ is lower than the maximin one (which is obtained by setting $\theta = 1$), and higher than the utilitarian one (obtained by setting $\theta = 0$).

Appendix 2: Further Notes on the Maximin Criterion

Many economists have criticized the maximin criterion. To them, this criterion implies an extreme form of risk aversion. In contrast, Strasnick (1976), using an axiomatic approach, argues that Rawls's formulation of the "original position" (when hypothetically individuals hiding behind the veil of ignorance meet to choose the principles of justice) necessarily implies the maximin principle[17].

We note in passing that, in practical applications, expressions such as "the worse off individual" should not be interpreted literally. According to Rawls (1999, p. 84), " for example, all persons with less than half of the median (income) may be regarded as the least advantaged segment."

A2. 1. Justifications of the maximin criterion in the absence of intergenerational considerations

Rawls advocated the maximin criterion in an "atemporal" (i.e., non-intergenerational) context. He postulated a hypothetical original position, in which the contracting parties are individuals hidden behind the veil of ignorance: none of them knows his place in society, his natural talents, intelligence, strength, and the like. (The individuals are facing *Knightian uncertainty*, which is totally different from a situation of choice under risk.) In other words,the principles of justice are agreed to in an initial situation that is fair. Rawls argued that the contracting parties would agree to two principles of justice. The first principle says that "each person is to have an equal right in the most extensive scheme of equal basic liberties compatible with a similar scheme of liberties for others"(1999, p. 53).The second principle states that social and economic inequalities are acceptable only if they are arranged so that they are "both (a) to the greatest expected benefit of the least advantaged and (b) attached

[17] His demontration relies on a set of axioms, the most important of which is Axiom 1 (Equal Priority Principle): For any four distributions $d, \widehat{d}$, $\widetilde{d}$ and $\overline{d}$, and any pair of individuals (A, B), if $\widehat{u}_A = \overline{u}_B$ then A's preference of d over $\widehat{d}$ must have the same priority as B's preference of $\widetilde{d}$ over $\overline{d}$.

to offices and positions open to all under conditions of fair equality of opportunity"(1999, p. 72). Rawls called the second principle "the difference principle," in preference to the more common term, "maximin."

In formulating the second principle, Rawls had in mind the fact that some degree of inequality may have an incentive effect that makes everyone better off. To quote: "To illustrate... consider the distribution of income among social classes...Now those starting out as members of the entrepreneurial class... have a better prospect than those who begin in the class of unskilled laborers. It seems likely that this will be true even when the social injustices which now exist are removed. What, then, can possibly justify this kind of initial inequality in life prospects? According to the difference principle, it is justifiable only if the difference of expectation is to the advantage of...the representative unskilled worker. The inequality in expectation is permissible only if lowering it would make the working class even more worse off...The greater expectation allowed to entrepreneurs encourages them to do things which raise the prospects of laboring class. Their better prospects act as incentives so that the economic process is more efficient, innovation proceeds at a faster space, and so on" (1999,pp. 67-68).

The tension between equity and efficiency noted by Rawls was well recognized by practitioners. As Lee Kwan Yew stated:

... "If performance and rewards are determined by the marketplace, there will be a few big winners, many medium winners, and a considerable number of losers. That would make for social tensions because a society's sense of fairness is offended....To even out the extreme results of free-market competition, we had to redistribute the national income...If we over-redistributed by higher taxation, the high performers would cease to strive. Our difficulty was to strike the right balance." (Lee, 2000, p. 116.)

A2.2. The maximin criterion for infinite utility sequences

The maximin criterion, which Rawls prefers to call "the difference principle", was advocated by Rawls only for choice among distributions for individuals that belong to the same generation. When dealing with intergenerational equity, Rawls argues that the difference principle must be modified.(I will explain later Rawls' arguments for a "modified difference principle" which he calls the "principle of just savings".)

Despite this fact, many economists have used the maximin criterion as an objective function for choosing among paths of consumption and investments for all generations. The value taken on by the social welfare function (under the maximin criterion) of a utility stream $\mathbf{u}^i$ is the value of the smallest u_t^i :

$$W^M(\mathbf{u}^i) = \inf_t \{u_t\}_{t=1,2,3,\dots}$$

Not surprisingly, this criterion, when applied to infinite sequences of utilities, where utility of an individual depends only on his consumption vector, implies that along the maximin path, the aggregate value of investments[18] is zero at each point of time[19]. Consider the standard neoclassical one-sector model. Suppose the initial capital stock k_1 is below the golden rule level k_G. Then consumption can be maintained at the level $c_1^* = f(k_1) - mk_1 > 0$ for ever[20]. Any attempt to raise future consumption would require current consumption to fall below c_1^*. Applying the maximin criterion, the "optimal choice" for society is that all generations must remain poor. This outcome, according to Rawls, offends our sense of justice.

A2. 3. Rawls' s criticism of the maximin criterion for utility sequences

Rawls has consistently argued that the maximin criterion, which he used as a central piece for his theory of justice among contemporaneous individuals, should not be applied to the choice among infinite sequences of utilities of

[18]See Hartwick (1977), Dixit el al. (1980), Hartwick and Long (1999), Long (2005). Here, it is assumed that all capital stocks are correctly priced, so that there is no divergence between "prices" and "values." Without correct pricing, an economist would be, in the words of Oscar Wilde (1926, Act III, p. 113), "someone who knows the price of everything and the value of nothing."

[19]Strictly speaking, this is true for continuous time models. For discrete time models, the aggregate value of investment is approximately zero along constant consumption paths.

[20]Here m is the rate of depreciation.

generations of individuals. (See Rawls, 1971, section 44, and 1999, pp. 255-7). To articulate a theory of justice between generations, Rawls suggests that one must modify the assumptions concerning the original position, by specifying that the the parties in the original position are "heads of families and therefore have a desire to further the well-being of at least their more immediate descendants"(1999, p. 111). Rawls also introduces the constraint that the just savings principle adopted must be such that the parties wish all earlier generations to have followed it.

Why is it necessary to bring such modifications to the difference principle? The answer is simple: the unmodifed difference principle would entail either no saving at all or not enough saving to improve social circumstances. Such a state of affairs would offend our sense of justice. In other words, one must modify assumptions so as to "achieve a reasonable result"(1999,p. 255).

The absence of savings is a concern for Rawls, especially if one is considering a society with a very low initial level of capital, because, "to establish effective just institutions within which the basic liberties can be realized"(1999,p.256), a society must have a sufficient material base. Generations must "carry their fair share of the burden of realizing and preserving a just society"(1999,p.257).

Rawls did not advance a specific criterion to replace his difference principle. He only made the point that the difference principle must be modified by a "principle of just savings". Such a principle should takes into account the fact that "fathers care for their sons". To quote Rawls (1971, p. 287-288):

"...The parties do not know which generations they belong to...They have no way of telling whether it is poor or relatively wealthy, largely agricultural or already industrialized....The veil of ignorance is complete in these respects. Thus the persons in the original positions are to ask themselves how much they would be willing to save at each stage of advance on the assumption that all other generations are to save the same rates...Since no one knows to which generation he belongs, the question is viewed from the standpoint of each...All the generations are virtually represented in the original position...Only those in the first generations do not benefit...for while they begin the whole process they do not share in the fruits of their provision. Nevertheless, since it is assumed that a generation cares for its immediate descendants, as fathers care for their sons, a just savings principle ...would be adopted."

While Rawls was of the opinion that early generations should save for the benefits of future generations, he thought that the utilitarian approach may require too much saving (1999, p.255)[21].

A2. 4 The maximin criterion with consumption-based parental altruism

Arrow (1973b) and Dasgupta (1974) were among the first economists who formulate a maximin criterion that incorporates Rawls's concerns for just savings. They postulated that the utility of generation t depends not only on their own consumption level, but also that of the next generation. Thus

$$u_t = u(c_t, c_{t+1})$$

We say that the function $u(c_t, c_{t+1})$ is a "utility function with consumption-based parental altruism" because the parents care about the *consumption vector* c_{t+1} of their offsprings, not about the latter's *utility level* (which would takes into account the utility level of their own future offsprings). Under this formulation, consider an economy starting at time $t = 0$. This economy must choose, among all possible paths of consumption, the one whose lowest u_t is greater than the lowest $u_{t'}$ of any other feasible path.

[21] It is interesting to note that Rawls's concern that too much sacrifice is imposed on the early generations had been voiced much earlier by a famous German philosopher, Immanuel Kant (1724-1804). In his essay, "Idea for a Universal History with a Cosmopolitan Purpose," Kant put forward the view that nature is concerned with seeing that man should work his way onwards to make himself worthy of life and well-being. He added: "What remains disconcerting about all this is firstly, that the earlier generations seem to perform their laborious tasks only for the sake of the later ones, so as to prepare for them a further stage from which they can raise still higher the structure intended by nature; and secondly, that only the later generations will in fact have the good fortune to inhabit the building on which a whole series of their forefathers...had worked without being able to share in the happiness they were preparing." See Reiss (1970, p. 44).

Dasgupta (1974) specializes in the separable form

$$u(c_t, c_{t+1}) = v(c_t) + \frac{1}{1+\delta} v(c_{t+1})$$

He assumes that the economy is capable of a constant growth rate $g > 0$:

$$k_{t+1} = (1+g)\left[k_t - c_t\right]$$

It turns out that applying the maximin criterion to the sequence $(u_1, u_2, ...)$ of this economy, if $\delta > g$, the economy will choose a constant consumption path. If $\delta < g$, the economy will experience cycles. As Dasgupta puts it, "The economy is not allowed to lift itself permanently out of poverty if it begins with a low value of k_0."(p.411).

Another troublesome feature of this model is that if when a new generation takes over the control of the economy, it will choose not to follow the path chosen by its predecessor. Thus we face the problem of "time inconsistency". One way out of this problem is to think of "just savings" not as what the first generation tells later generations to do, but as what each generation would do, taking into account that each successor will do what it will deem "just". In the game-theoretic language, one should be looking for a Nash equilibrium sequence of saving rates. Dasgupta was able to characterize such a Nash equilibrium in his simple model. Unhappily, in that equilibrium, the utility sequence is not Pareto efficient.

A2. 5 The maximin criterion with utility-based parental altruism

It is arguable that parents should care about their offsprings's utility (rather than the latter's consumption vector) which in turn depends on the utility of the latter's offsprings. Maximin can then be applied to such a stream of utilities. This is the approach adopted by Calvo (1978), Rodriguez (1981), and Asheim (1988). Let ${}_tc$ denote a sequence of consumption starting from time t, and c_t the consumption at time t. Assume

$$u({}_tc) = \sum_{s=t}^{\infty} \beta^{s-t} v(c_s) = v(c_t) + \beta u({}_{t+1}c) \text{ where } 0 < \beta < 1$$

The social welfare function under the maximin criterion is

$$w({}_tc) = \inf_{s \geq t} u({}_sc)$$

Applying this criterion to the simple one-sector model, Calvo (1978) and Rodriguez (1981) show that the optimal program is time-consistent. Unfortunately, when the model is extended to have a non-renewable resource, Asheim (1988) finds that time-inconsistency reappears.

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