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Forecasting U.S. Recessions and Economic Activity*

Rachidi Kotchoni†, *Dalibor Stevanovic*‡

Abstract

This paper proposes a framework to produce multi-horizon forecasts of business cycle turning points, average forecasts of economic activity as well as conditional forecasts that depend on whether the horizon of interest belongs to a recession episode or not. Our forecasting models take the form of an autoregression (AR) of order one that is augmented with either a probability of recession or an inverse Mills ratio. Our empirical results suggest that a static Probit model that uses only the TS as regressor provides comparable fit to the data as more sophisticated non-static Probit models. We also find that the dynamic patterns of the term structure of recession probabilities are quite informative about business cycle turning points. Our most parsimonious AAR model delivers better out-of-sample forecasts of GDP growth than the benchmark models considered. We construct term structures of recession probabilities since the last oficial NBER turning point. The results suggest that there has been no harbinger of a recession for the US economy since 2010Q4 and that there is none to fear at least until 2018Q1. GDP growth is expected to rise steadily between 2016Q3 and 2018Q1 in the range [2.5%,3.5%].

Keywords: Augmented Autoregressive Model, Conditional Forecasts, Economic Activity, Inverse Mills Ratio, Probit, Recession

Codes JEL/JEL Codes: C35, C53, E27, E37

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1 Introduction

The economic activity is characterized by swings across peaks and troughs over time. A period running between a peak and the next trough is called a *recession* while a period between a trough and the next peak is an *expansion*. Episodes during which the level of economic activity remained roughly constant are referred to as *stagnation*. Such episodes are generally amalgamated with expansions under the label *"no recession."* By abuse of language and for the sake of simplicity, we often refer to the "no recession" periods as expansion periods.

The definition of a recession given above raises two practical issues. The first issue concerns the precise meaning of the expression "*economic activity*". The Business Cycle Dating Committee of the National Bureau of Economic Research (NBER) does not provide a precise definition to this expression. Rather, it defines a recession as "*a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales.*" The second issue concerns the identification of the business cycle turning points (i.e., peaks and troughs of real economic activity) from the observed data. The Business Cycle Dating Committee does provide a precise response to the latter issue by regularly publishing recession dates with approximately one year lag[1].

This paper proposes a framework to produce multi-horizon real time predictions of the probability of a recession, forecasts of average economic activity (typically, the GDP growth) and conditional forecasts that depend on whether the horizon of interest belongs to a "recession" episode or not. Our workhorse is an Augmented Autoregressive (AAR) model, which is a simple direct autoregressive (AR) model of order one augmented with probabilities of recession and/or inverse Mills ratios (IMR). The probability of a recession at a given horizon h is predicted conditionally on time t information using a simple static Probit model. A forecast that is conditional on an expansion at the horizon of interest is termed optimistic

[1] The business cycle dates can be found at http://www.nber.org/cycles.html.

and a forecast done conditionally on a recession scenario is termed pessimistic. Methods are also proposed to infer the business cycle turning points from the dynamics of the term structure of recession probabilities.

Optimistic and pessimistic forecasts can in principle be obtained by splitting the sample according to whether there is a recession or not, as done for example in (Hamilton 2011). Here, we follow an alternative approach that involves IMR corrections. (Dueker 2005) and (Dueker & Wesche 2005) proposed a Qual VAR model, which is a VAR system that includes a latent variable that governs the occurrence of a binary outcome. This approach is not favored here because it does not naturally lead to state-dependent forecasts of economic activity in real time.[2]

Our AAR model falls within the broad family of conditional forecasting models studied by (Clark & McCracken 2013). This family includes all forecasting models that assume a particular policy path (e.g., announced inflation target) or a scenario for the future path of a given macroeconomic variables (e.g., low inflation and high unemployment)[3]. Conditional forecasting models are used by major financial institutions and regulatory agencies worldwide in order to perform stress tests, see e.g. (Grover & McCracken 2014). Typically, the goal of a stress testing exercise is to predict the impact of a more or less strong adverse shocks affecting one sector or the overall business environment on a particular outcome. The methodology developed in this paper can be useful in that context. Indeed, our pessimistic forecast can serve as input for a wide range of stress testing models.

Our empirical application starts with a set of *in-sample* performance evaluation exercises. First, we compare the in-sample performance of our static Probit model to that of three

[2]Our paper assumes that recession dates are observed up to the most recent official turning point. Studies that attempt to predict the business cycle turning points include: [(Chauvet 1998), (Chauvet & Hamilton 2006), (Chauvet & Piger 2008), (Stock & Watson 2010), (Berge & Jorda 2011) (Stock & Watson 2012) or (Ng 2014)]. Other studies attempt to identify the variables that lead future economic activity, e.g. [(Stock & Watson 1989), (Issler & Vahid 2006), (Ng & Wright 2013)].

[3]For instance, (Giannone, Lenza, Momferatou & Onorante 2010) perform an inflation forecasting exercise conditional on pre-specified paths for oil price indicators. (Schorfheide & Song 2013) produce inflation and growth forecasts conditional on forecasts obtained from judgmental sources. Other references on conditional forecasts include (Sims 1982), (Doan, Litterman & Sims 1984), (Meyer & Zaman 2013) and (Aastveit, Carriero, Clark & Marcellino 2014).

dynamic Probit models and two Markov Switching (MS) models. The results suggest that the dynamic specifications outperform the static Probit model only at horizons 1 and 2. The MS models predict recessions very well in-sample but as we shall see later, they perform poorly out-of-sample. Second, we estimate the static Probit model for the probability of recession using three conditioning information sets. The first information set contains the term spread only, the second contains the term spread and the federal fund rate while the third further contains the credit spread. We find that the Probit model that uses only one regressor (i.e., the term spread) compares favorably to those that use two or more regressors, especially at horizons $h = 3$ and beyond. Third, we compare the AAR to a simple AR model, an Augmented Distributed Lag (ADL) model and a MS model in terms of their ability to forecast GDP growth a few quarters ahead. The ADL model used is a version of the AAR model that implicitly assumes a linear structure for the probability of a recession. The AR model produces uninformative forecasts as soon as the horizon exceeds $h = 3$. The MS model does well at horizons 1 and 2 but its performance deteriorates fast as h increases. The ADL model performs better than the AR model but is less and less resilient than the AAR model as the forecast horizon increases.

We use our model to conduct a real time out-of-sample analysis of the last four recessions of the US economy. First, we estimate a static Probit model for the probability of a recession and an AAR model for the GDP growth using a sample that stops at the latest official turning point before each recession. Second, the estimated parameters and the most recent release of GDP are plugged into the AAR model to obtain forecasts of the probability of a recession and of GDP growth rate at different horizons. Finally, an ADL model and a MS model are estimated and their out-of-sample predictions compared to those of the AAR model. Our results suggest that the dynamic patterns of the term structure of recession probabilities are informative about business cycle turning points. We also find that the AAR model delivers more accurate real time forecasts than does the AR, ADL and MS models.

The remainder of the paper is organized as follows. Section 2 details the construction of the AAR model. Section 3 motivates the static Probit model used for the probability of recessions and discusses alternative approaches. Section 4 presents our strategy to infer turning points from the term structure of the probability of recession in real time. Sections 5 and 6 present the empirical application and prospects since the last NBER announcement, and section 7 concludes. Supplementary material are available in a separate document.

2 Modeling the Economic Activity

Let y_t denote an economic activity variable (e.g., GDP growth, unemployment rate, etc.), $R_t \in \{0,1\}$ the indicator of recession[4] at time t and X_t a set of potential predictors of recessions. Our main objective is to produce multi-horizon forecasts of the variables y_t. For that purpose, we consider using a family of Augmented AutoRegressive (AAR) models specified at a quarterly frequency.

The intended models normally take the form:

$$y_{t+h} = \rho_{h,0} + \rho_{h,1} y_t + \delta_h R_{t+h} + v_{t+h}, \tag{1}$$

for $t = 1, ..., T-h$, where $h \geq 1$ is the forecast horizon and $v_{t+h} \sim N(0, \sigma_h^2)$ is a Gaussian error term. This error is assumed potentially correlated with R_{t+h} but uncorrelated with lagged realizations of y_t. Unfortunately, these models cannot be used for real time forecasting as the right hand side contains a regressor that is not yet observed at period t.

Taking the expectation of y_{t+h} conditional on the information available at time t yields:

$$E\left(y_{t+h}|y_t, X_t\right) = \rho_{h,0} + \rho_{h,1} y_t + \delta_h \Pr\left(R_{t+h} = 1|y_t, X_t\right).$$

For the sake of simplicity, the probability of a recession at time $t+h$ is assumed to depend on X_t only. Furthermore, we assume that the likelihood of a recession obeys a Probit model.

[4]That is, $R_t = 1$ if the NBER dating committee designated period t as a recession time and $R_t = 0$ otherwise.

This leads to:

$$\Pr\left(R_{t+h}=1|y_t,X_t\right)=\Pr\left(R_{t+h}=1|X_t\right)=\Phi\left(X_t\gamma_h\right), \tag{2}$$

where Φ is the cumulative distribution function (CDF) of the standard normal random variable. Therefore, an equation that expresses the expected value of y_{t+h} in terms of quantities that depends on the information available at time t is given by:

$$E\left(y_{t+h}|y_t,X_t\right)=\rho_{h,0}+\rho_{h,1}y_t+\delta_h\Phi\left(X_t\gamma_h\right)\equiv\widehat{y}_{t+h}, \tag{3}$$

Accordingly, y_{t+h} may be represented as an Augmented Autoregressive (AAR) process as follows:

$$y_{t+h}=\rho_{h,0}+\rho_{h,1}y_t+\delta_h\Phi\left(X_t\gamma_h\right)+\widetilde{v}_{t+h}, \tag{4}$$

where $\widetilde{v}_{t+h}\equiv v_{t+h}+\delta_h\left(R_{t+h}-\Phi\left(X_t\gamma_h\right)\right)$ is a zero mean error term.

Note that the forecasting formula (3) exploits the information content of X_t in a nonlinear manner. This suggests an alternative ADL model where X_t enters linearly in the right hand side. That is:

$$y_{t+h}=\rho_{h,0}+\rho_{h,1}y_t+X_t\beta_h+\varepsilon_{t+h}, \tag{5}$$

where ε_{t+h} is an error term. The model above implicitly assumes a linear structure for the probability of a recession. ADL models similar to this have been explored, among others, in (Gilchrist, Yankov & Zakrajsek 2009), (Chauvet & Potter 2013) and (Ng & Wright 2013). The valued added of the AAR model (4) vis-à-vis the ADL model (5) is attributable to nonlinearity.

It is possible to go one step beyond the average forecast (3) by taking advantage of the observability of the recession dates (R_t). Indeed, a forecast can be generated based on the pessimistic scenario that the economy will experience a recession at horizon $t+h$, i.e.:

$$E\left(y_{t+h}|y_t,X_t,R_{t+h}=1\right)=\rho_{h,0}+\rho_{h,1}y_t+\overline{\delta}_h+\delta_{h,1}E\left(\widetilde{v}_{t+h}|y_t,X_t,R_{t+h}=1\right).$$

Under a joint Gaussianity assumption on v_{t+h} and the error term of the latent equation underlying the Probit (2), we obtain:

$$E\left(y_{t+h}|y_t, X_t, R_{t+h}=1\right) = \rho_{h,0} + \rho_{h,1}y_t + \overline{\delta}_h + \delta_{h,1}\frac{\phi\left(X_t\gamma_h\right)}{\Phi\left(X_t\gamma_h\right)} = \underline{y}_{t+h}, \tag{6}$$

where $\delta_{h,1} = Cov\left(u_{h,t}, v_{t+h}|R_{t+h}=1\right)$, ϕ is the probability distribution function (PDF) of the standard normal random variable, $\overline{\delta}_h$ is the intrinsic effect of the recession and $\delta_{h,1}\frac{\phi(X_t\gamma_h)}{\Phi(X_t\gamma_h)}$ stems from a "break" in the structure of dependence between y_{t+h} and X_t due to the recession. Note that this break is absent when v_{t+h} is uncorrelated with recessions so that $\delta_{h,1} = 0$. The pessimistic forecast may be used to assess how severe a recession is expected to be if it were to effectively occur at the forecast horizon. This kind of formula can be used to perform a wide range of stress testing exercise in the banking sector, anticipate extreme losses on a portfolio, assess the fragility of the housing sector, etc.

Likewise, another forecast based on the optimistic scenario of no recession at horizon $t+h$ can be computed as:

$$E\left(y_{t+h}|y_t, X_t, R_{t+h}=0\right) = \rho_{h,0} + \rho_{h,1}y_t + \delta_{h,0}\frac{-\phi\left(X_t\gamma_h\right)}{1-\Phi\left(X_t\gamma_h\right)} = \overline{y}_{t+h}, \tag{7}$$

where $\delta_{h,0} = Cov\left(u_{h,t}, v_{t+h}|R_{t+h}=0\right)$ and $\delta_{h,0}\frac{\phi(X_t\gamma_h)}{1-\Phi(X_t\gamma_h)}$ is a break that marks expansion periods. This optimistic forecast can be used to assess how favorable the economic conjuncture is expected to be if an expansion were to occur at the forecast horizon.

The variables $\frac{\phi(X_t\gamma_h)}{\Phi(X_t\gamma_h)}$ and $\frac{-\phi(X_t\gamma_h)}{1-\Phi(X_t\gamma_h)}$ are the well-known inverse Mills ratios (IMR). The parameters $\overline{\delta}_h$, $\delta_{h,0}$ and $\delta_{h,1}$ are all expected to be negative if y_t is pro-cyclical (that is, if y_t increases during expansions and shrinks during recessions). In our framework, the terms $\delta_{h,1}\frac{\phi(X_t\gamma_h)}{\Phi(X_t\gamma_h)}$ and $\delta_{h,0}\frac{-\phi(X_t\gamma_h)}{1-\Phi(X_t\gamma_h)}$ capture the combined effects of things that are hard to measure such as supply and demand shocks, policy responses to these shocks, investors sentiments, consumer confidence, agents anticipations, etc.

Pooling the forecasting formulas (6) and (7) yields:

$$y_{t+h} = \rho_{h,0} + \rho_{h,1} y_t + \overline{\delta}_h R_{t+h} + \delta_{h,0} IMR_{t,h,0} + \delta_{h,1} IMR_{t,h,1} + \widetilde{\widetilde{v}}_{t+h}, \tag{8}$$

where $\widetilde{\widetilde{v}}_{t+h}$ is a zero mean error term and:

$$IMR_{t,h,1} = \begin{cases} \frac{\phi(X_t\gamma_h)}{\Phi(X_t\gamma_h)} & \text{if } R_{t+h}=1, \\ 0 & \text{otherwise.} \end{cases},$$

$$IMR_{t,h,0} = \begin{cases} \frac{-\phi(X_t\gamma_h)}{1-\Phi(X_t\gamma_h)} & \text{if } R_{t+h}=0, \\ 0 & \text{otherwise.} \end{cases}.$$

To implement the AAR model empirically, we first estimate a Probit model for the probability of recessions to obtain $\widehat{\gamma}_h$. This estimate is used to compute fitted values for the probability of recession $\widehat{P}_{t,h} = \Phi\left(X_t\widehat{\gamma}_h\right)$ and for the inverse Mills ratios $\widehat{IMR}_{t,h,1}$ and $\widehat{IMR}_{t,h,0}$. The average forecasts are obtained as the fitted values of the following OLS regression:

$$y_{t+h} = \rho_{h,0} + \rho_{h,1} y_t + \delta_h \widehat{P}_{t,h} + e_{t+h}, \tag{9}$$

where e_{t+h} is an error term. Finally, the parameters used to compute the optimistic and pessimistic forecasts are deduced from the following OLS regression:

$$y_{t+h} = \rho_{h,0} + \rho_{h,1} y_t + \overline{\delta}_h \widehat{P}_{t,h} + \delta_{h,0} \widehat{IMR}_{t,h,0} + \delta_{h,1} \widehat{IMR}_{t,h,1} + \widetilde{e}_{t+h}, \tag{10}$$

where $\widetilde{e}_{t+h}$ is an error term. Recall that R_{t+h} is replaced by $\widehat{P}_{t,h}$ above as a means to avoid endogeneity biases.

An alternative framework to forecasting economic activity conditionally on the state of economy is provided by the Markov Switching (MS) model of (Hamilton 1989). The simplest version of this model allows only the intercept to be state-dependent, that is:

$$y_{t+h} = \mu_{R_t} + \rho y_t + \varepsilon_{t+h} \tag{11}$$

where $\varepsilon_{t+h} \sim N(0, \sigma_\varepsilon^2)$. In a more flexible specification, the autoregressive root is allowed to be state-dependent as well:

$$y_{t+h} = \mu_{R_t} + \rho_{R_t} y_t + \varepsilon_{t+h} \tag{12}$$

It is further possible to make ε_{t+h} heteroskedastic by letting its variance depend on R_t. However, we restrict (11) and (12) to the homoskedastic case in our empirical applications.

Note that our AAR model is comparable to a regime switching model in some dimensions. The key differences are that the AAR model treats R_t as observed (rather than latent) and it specifies R_t as a function of other lagged explanatory variables (rather than its own lag).

3 Modeling the Probability of Recession

In the previous section, we have chosen to model the probability of a recession using a static Probit for three reasons. First, this model has a structural flavor as it naturally emerges from assuming the existence of a latent lead indicator $Z_{h,t}$ that takes the form:

$$Z_{h,t} = X_t \gamma_h + u_{h,t}, \text{ for all } t \text{ and } h, \tag{13}$$

with $u_{h,t} \sim N(0, 1)$, and which satisfies:

$$R_{t+h} = \begin{cases} 1 & \text{if } Z_{h,t} > 0, \\ 0 & \text{otherwise.} \end{cases} . \tag{14}$$

Second, our optimistic and pessimistic forecasting formulas depends on the expressions of $E\left(v_{t+h}|X_t, R_{t+h}\right)$, for $R_{t+h} \in \{0, 1\}$. These expressions are easily calculated by assuming that $(u_{h,t}, v_{t+h})$ are jointly Gaussian, for all $h \geq 1$.[5] The third argument in favor of the static Probit model is its simplicity.

The IMR terms resulting from the calculation of $E(v_{t+h}|X_t, R_{t+h}), R_{t+h} \in \{0,1\}$ have the usual interpretation of Heckman (1979)'s sample selection bias correction. Indeed, the Probit model (13)-(14) may be viewed as an attempt to infer the behavior of the NBER dating committee from historical data. The AAR model attempts to capture patterns in the data that prompted the NBER committee to label certain dates as recessionary and others as expansionary. Business cycle turning points are announced with lags of up to four quarters. As suggested by (Wright 2006), a probabilistic model like the one above is interesting in its own as it can be trained on historical data and used to infer the next tuning point pending an NBER official announcement.

The exercise which consists of predicting the probability of recessions is not new in the literature. (Stock & Watson 1989) used a probabilistic framework to construct a coincident and a leading index of economic activity as well as a recession index. (Estrella & Mishkin 1998) examined the individual performance of financial variables such as interest rates, spreads, stock prices and monetary aggregates at predicting the probability of a recession. They found that stock prices are good predictors of recessions at one to three quarters horizon while the slope of the yield curve is a better predictor beyond one quarter. The forecasting power of the yield curve is also documented in (Rudebusch & Williams 2009), who find that professional forecasters do not properly incorporate the information from the yield spread. (Nyberg 2010) advocated a dynamic probit model and found that in addition to term spread (TS), lagged values of stock returns and foreign spreads are important predictors of a recession. (Anderson & Vahid 2001) applied nonlinear models to predict the probability of U.S. recession using the interest-rate spread and money stock (M2) growth[6]. (Wright 2006)

[5]Note that it is not clear how one would generate conditional forecasts analogue to (6) and (7) in the context of the ADL model.

[6]These authors use (Fair 1993)'s definition of a recession. (Fair 1993) defines a recession as either "at least two consecutive quarters of negative growth in real GDP over the next five quarters" or "at least two quarters of negative growth in real GDP over the next five quarters." This definition is not retained by the NBER.

estimated several Probit models and found that adding the federal funds rate (FFR) to the term spread outperforms the model of (Estrella & Mishkin 1998) that used the term spread only. Recently, (Christiansen, Eriksen & Mø ller 2013) found that sentiment variables have predictive power beyond standard financial series.

(Kauppi & Saikkonen 2008) considered three different dynamics for the latent lead indicator $Z_{h,t}$. The first specification below leads to the simplest *Dynamic Probit* model:

$$Z_{h,t+h} = X_t \gamma_{1,h} + R_t \gamma_{2,h} + u_{h,t}, \text{ for all } t. \tag{15}$$

The second specification, the *Autoregressive Probit* model, is given by:

$$Z_{h,t+h} = X_t \gamma_{1,h} + Z_{h,t} \gamma_{2,h} + u_{h,t}, \text{ for all } t. \tag{16}$$

Finally, the third specification gives rise to the *Dynamic Autoregressive Probit* model:

$$Z_{h,t+h} = X_t \gamma_{1,h} + R_t \gamma_{2,h} + Z_{h,t} \gamma_{3,h} + u_{h,t}, \text{ for all } t. \tag{17}$$

These specifications incorporate the inertia of $Z_{h,t}$ and R_t when forecasting future recessions.

(Hao & Ng 2011) have found that dynamic Probit models improve upon the static Probit, especially when predicting the duration of recessions. This is of course expected, in particular for short term horizons and around turning points. However, the dynamic feature of these models makes them unsuitable for a real-time forecasting exercise as R_t is usually observed with at least one-year lag. A static Probit that uses financial predictors released at high frequency does not suffer from this shortcoming.

4 Predicting Turning Points in Real Time

There is a difference between the prediction of the probability of a recession and the prediction of the beginning and end of a recession. The latter exercise is slightly more difficult as it requires decision science tools in addition to a probabilistic model. This section discusses how to infer turning points from the predicted probabilities of the recession in real time.

At a quarterly frequency, the first release of GDP is available with one lag while the 'final' value is released with approximately one year lag.[7] The NBER turning points are released with at least four lags. These aspects may be ignored if we are interested only in assessing the *in-sample* performance of the Probit and AAR models based on historical data. However, a strategy to deal with release lags is needed if one wishes to conduct a real time analysis.

If the current period is t^* and the latest turning point occurred at period $t^* - l$, then the final releases of NBER recession dates are available in real time only up to period $t^* - l$. Therefore, we can use fully revised data covering the periods $[1, t^* - l]$ to estimate the probability of recessions at any horizon h. We have:

$$\Pr\left(R_{t+h} = 1 | X_t\right) = \Phi\left(X_t \gamma_h\right),\ t \in [1, t^* - l - h].$$

The estimate $\widehat{\gamma}_h$ of γ_h obtained from above can be used to generate out-of-sample forecasts of the probability of recession. As we choose to include only high frequency financial variables in X_t, this out-of sample exercise does not suffer from release lag problems. We therefore can compute $\widehat{P}_{t,h} = \Phi\left(X_t \widehat{\gamma}_h\right)$ as well as the variables $\widehat{IMR}_{t,h,0}$ and $\widehat{IMR}_{t,h,1}$ for periods $t \in [1, t^*]$. Note that the out-of-sample periods runs from $t^* - l - h + 1$ to t^* for this Probit.

The next step is to estimate the AAR model for an economic activity variable based on the available information. At a quarterly frequency, the first release of economic activity variables is generally available with only one lag. Nonetheless, we constrain the in-sample

[7]In the realm of real time data, the "final value" of a variable is a release that is unlikely to be revised in the future. Strictly speaking, there is actually no final value.

period to be the same as for the Probit model. That is, we estimate Equations (9) and (10) by OLS using the sample covering the periods $t \in [1, t^* - l - h]$. At time t^*, the most recent release of the GDP growth is for the period $t^* - 1$. Equations (3), (6) and (7) take this latest release and the estimates $\widehat{\gamma}_h$ as input to return nowcasts ($h = 1$) and forecasts ($h > 1$) of economic activity. A similar strategy is employed for the ADL and MS models.

Using our static Probit model, we can calculate the term structure of the probability of recession at a given period t as the mapping $P_t : h \mapsto \Phi\left(X_t \widehat{\gamma}_h\right)$, $h \geq 1$. As we move forward from period t to periods $t+1$, $t+2$, etc., the term structure of recession probabilities is updated to P_{t+1}, P_{t+2}, etc. Our empirical experiments show that the sequence $P_t, t > 1$ is clustered into successive blocs of convex and concave curves. This suggests two possible strategies to identify turning points.

The first strategy relies on the upper envelope of the concave blocks and the lower envelope of the convex blocs. Suppose that at period t the term structure of recession probabilities P_t is concave. At that period, the next business cycle peak is predicted to occur at $t + h_t$, where h_t is the horizon where P_t is maximized. If P_t, P_{t+1}, ..., P_{t+H}, $H \geq 1$ is a block of concave term structure of recession probabilities, then we can compute an upper envelope curve for this bloc and predict the beginning of the next recession as the maximum of this curve. Business cycle troughs are predicted similarly. If P_t is convex, then the next business cycle trough is expected to occur at $t + \tau_t$, where τ_t is the horizon that minimizes P_t. Considering a bloc P_t, P_{t+1}, ..., $P_{t+L}, L \geq 1$ of convex term structure of recession probabilities, we can compute a lower envelope curve for this bloc and predict the end of the next recession as the minimum of this curve.

The second strategy relies on the timing of the changes in the shape of the term structure of recession probabilities. Indeed, a convex term structure curve of recession probabilities suggest that recession is less and less likely for some time. If this curve suddenly switches from convex to concave, this suggests that a new signal that raises the prospects of a recession just came in. One might therefore want to predict the beginning of the next recession as

$t+h_t$, where h_t is the horizon that maximizes P_t, and P_t is the beginning of a concave block. Likewise, the end of a recession may be predicted as $t+\tau_t$, where τ_t is the horizon that minimizes P_t and P_t is the beginning of a convex block.

With these two complementary strategies to identify turning points in hand, we analyse below the last four recessions that occurred in the US in order to see whether they were predictable ex ante.

5 Empirical Application

For this application, we use the quarterly NBER recession indicator available in the FRED2 database. Data on term spread (TS), credit spread (CS) and federal funds rate (FFR) are also obtained from the same source.[8] The real time vintages of GDP data are obtained from ALFRED database of the St. Louis Fed. The time span starts in 1959Q1 and ends in 2015Q2. We consider three different designs for X_t. In the first design, X_t is restricted to contain TS only. In the second design, X_t contains TS and CS. In the third design, X_t contains TS, CS as well as FFR. It is found that the addition of CS and FFR to TS generally adds little to the predictive power of our Probit models, especially at horizons above $h=2$ quarters. Therefore, most of the results presented in the main text are for the case where X_t reduces to TS. Additional results are available as supplementary material.

5.1 Full-Sample Analysis

This section presents in-sample forecasts based on models that are estimated on the full sample. The sample used here consists of historical data and release lag issues are ignored.[9]

[8] The term spread is defined as difference between 10-Year Treasury Constant Maturity Rate (labelled GS10 in FRED2) and 3-Month Treasury Bill: Secondary Market Rate (TB3MS). The credit spread (BAA10YM) is Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity.

[9] Hence, the full sample ends on 2014Q1 and contains only the most recent values of GDP, as obtained from the 2015Q2 vintage.

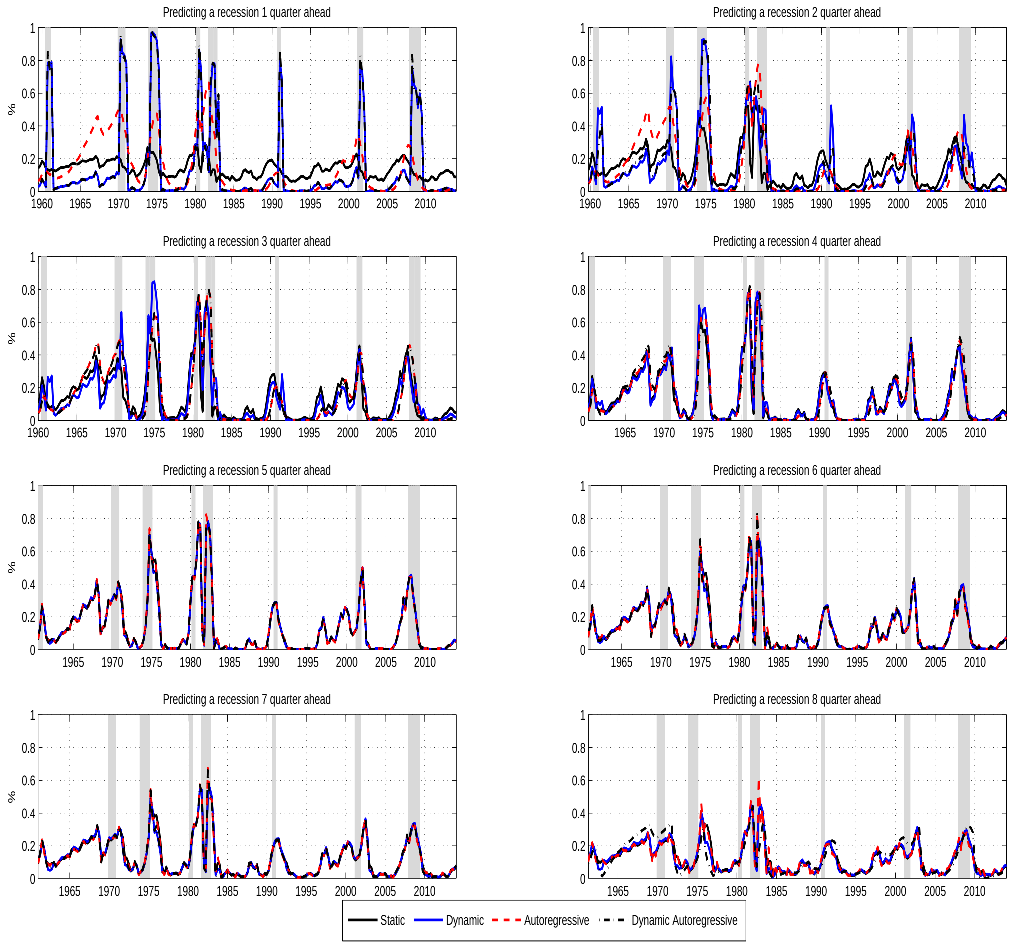
5.1.1 Static vs. Dynamic Probit Models

Figure 1 compares the in-sample fits of the static Probit model (2) to those of the dynamic Probit models (15), (16) and (17). All models are estimated using the term spread only. At horizons 1 and 2, the static Probit model predicts lower probabilities of recession during recession times and higher probabilities of recession during expansion times than the dynamic Probit models. At longer horizons, the advantage of the dynamic Probit models erodes so that there is no visible difference between the performances of all four models. These results supports that the dynamic Probit models provide a more reliable signal only when recession is imminent. This is expected because recession episodes do not last enough to allows full exploitation of the potential of the dynamic Probit models at the longer forecast horizons.

Figure 2 compares the performance of the static Probit model across different conditioning information sets. As explained previously, the first Probit model is estimated using TS only as regressor. The second model is estimated using TS and FFR while the third model combines TS, CS and FFR. The figure also shows the filtered probabilities of recession predicted by the MS model with state-dependent intercept. We see that these filtered probabilities fit the data better than the Probit models at all horizons. However, this impressive in-sample capability of the MS model is generally deceptive about its out-of sample performance.

At short horizons, the addition of FFR and CS seems to improve the performance of the static Probit model. This is particularly visible around 1965 where there was no recession while the probability of recession predicted using TS only is higher than the ones predicted based on the two other information sets. However, the predictions of all three static Probit models become more and more similar as the forecasting horizon increase. If we abstract from the beginning of the sample, the differences between the predicted probabilities of the Probit models are indeed negligible at horizons $h \geq 3$.

Figure 1: Predicting the probability of a recession: In-sample performance of static and non-static Probit models



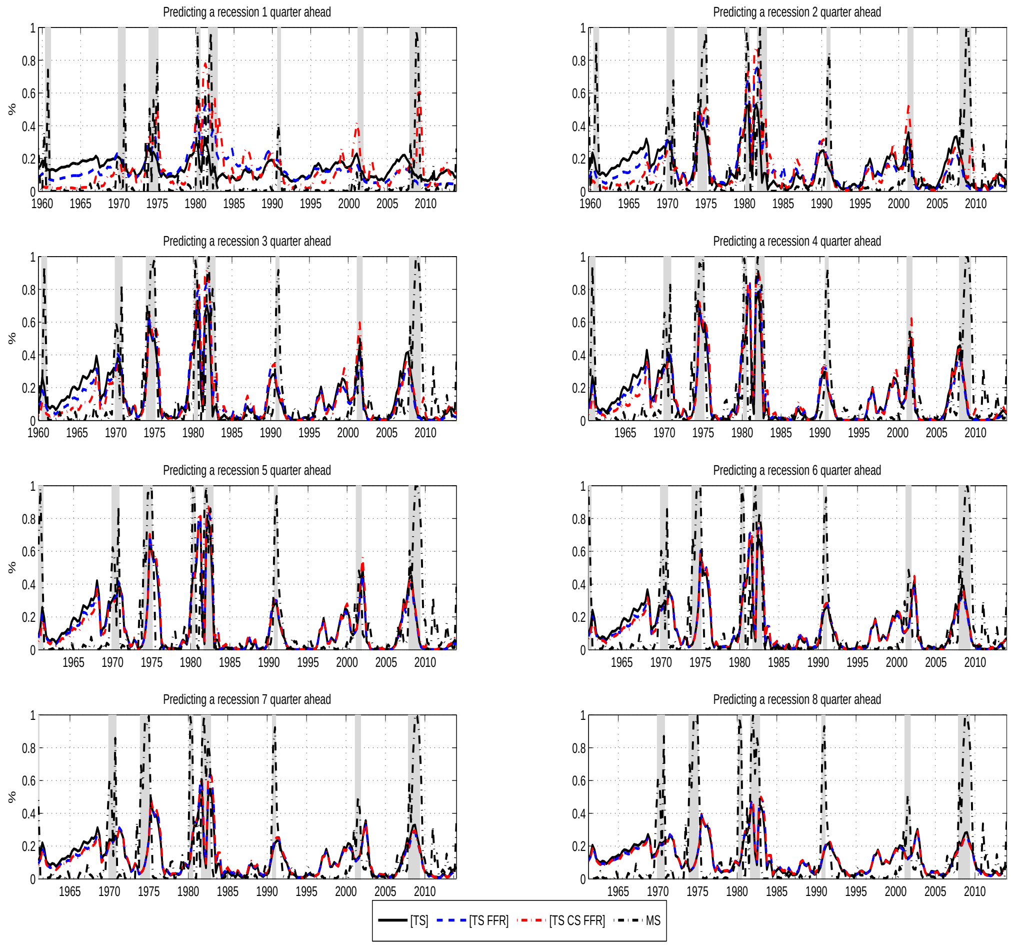
This figure shows in-sample predictions of the probability of a recession obtained from the static, dynamic, autoregressive and dynamic-autoregressive Probit models with the Term Spread only as regressor.

Overall, the results shown of Figures 1 and 2 suggest that the static Probit model estimated with TS alone is a good benchmark, especially if our focus is on horizons 3 quarters and beyond. The latter model is therefore favored because of its parsimony and its convenience for real time analysis.

5.1.2 AAR vs. AR, ADL and MS models

We now compare the performance of the AAR model to that of the benchmark models (namely the AR, ADL and MS) at predicting GDP growth. Figure 3 shows the adjusted R-squares of the AR, ADL and AAR models on the left vertical axis and the Student t-stat

Figure 2: Predicting the probability of a recession: In-sample performance of the static Probit model across different information sets

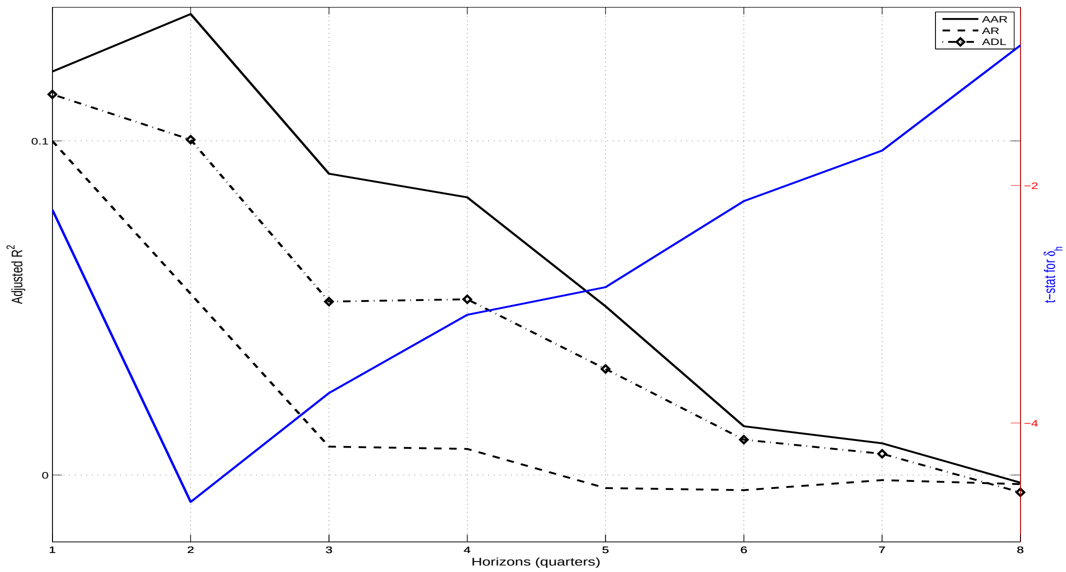


This figure shows in-sample predictions of the probability of a recession obtained from the static Probit model conditioned on three different information sets: [TS], [TS FFR] and [TS FFR CS]. MS stands for the filtered probabilities obtained from the Markov Switching model with state-dependent intercept.

associated with $\widehat{\delta}_h$ on the right vertical axis. Recall that the AAR models reduces to an AR model when $\delta_h = 0$. We see that the adjusted R-square is larger for the AAR model than for the AR at horizons $h = 1$ to $h = 7$. Accordingly, the parameter δ_h is estimated to be significant for these lags (i.e., t-stat larger than 2 in absolute value). The gap between the adjusted R-squares of the two models decreases with h and the AR model underperforms the historical average at lags beyond $h = 4$.

The ADL model fits that data better than the AR model but underperforms the AAR model. This suggests that the nonlinear transformation applied to X_t prior to its inclusion in the right hand side of Equation (4) matters. Putting it differently, the probability of recession at a given horizon is a relevant predictor of GDP growth at that horizon.

Figure 3: Predicting GDP growth: In-sample goodness-of-fit of the AR, AAR and ADL models

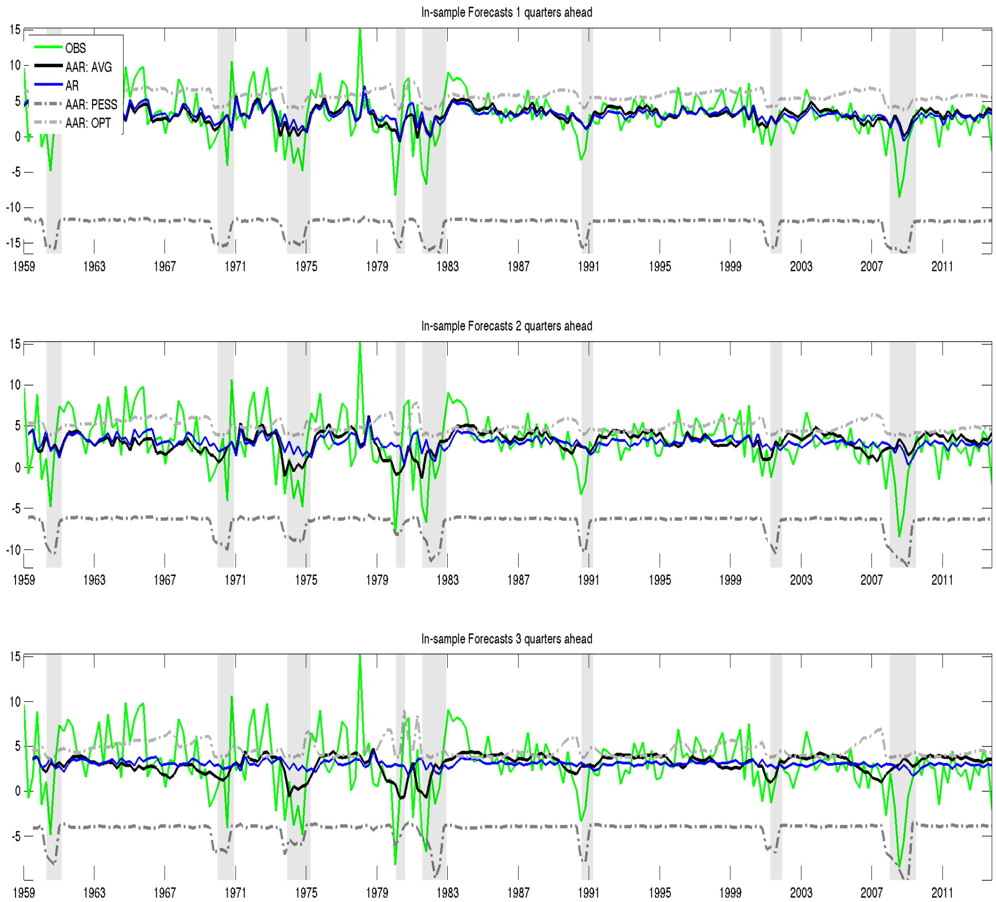


This figure shows the adjusted R^2 of the AAR, AR and ADL models on left vertical axis (full black line and dotted lines respectively), and the Student t-stat associated with $\widehat{\delta}_{i,h}$ on the right vertical axis. The probabilities of recession have been estimated from the static Probit model conditioned on TS only.

Figures 4 and 5 show the in-sample fit of the AR and AAR models as well as the optimistic and pessimistic forecasts. We note that the trajectory of fitted values produced by the AR model becomes roughly flat beyond horizon $h = 3$. At glance, the AAR model clearly dominates the AR model at horizons $h \geq 3$. This is consistent with the term structure of adjusted R-squares seen on Figure 3. The optimistic forecast is not optimistic enough as it often falls below the actual realizations of GDP growth. The pessimistic scenario is too pessimistic at horizons 1 and 2 and more realistic at longer horizons. One possible explanation of this asymmetric pattern is that the state $R_t = 0$ is actually a "no recession" state, which merges both stagnation and expansion.

Figures 6 and 7 compare the trajectory of fitted values for the AAR and MS models. First, we note that the MS model that allows both the intercept and the autoregressive root to be state-dependent does not outperform its more parsimonious version that constrains

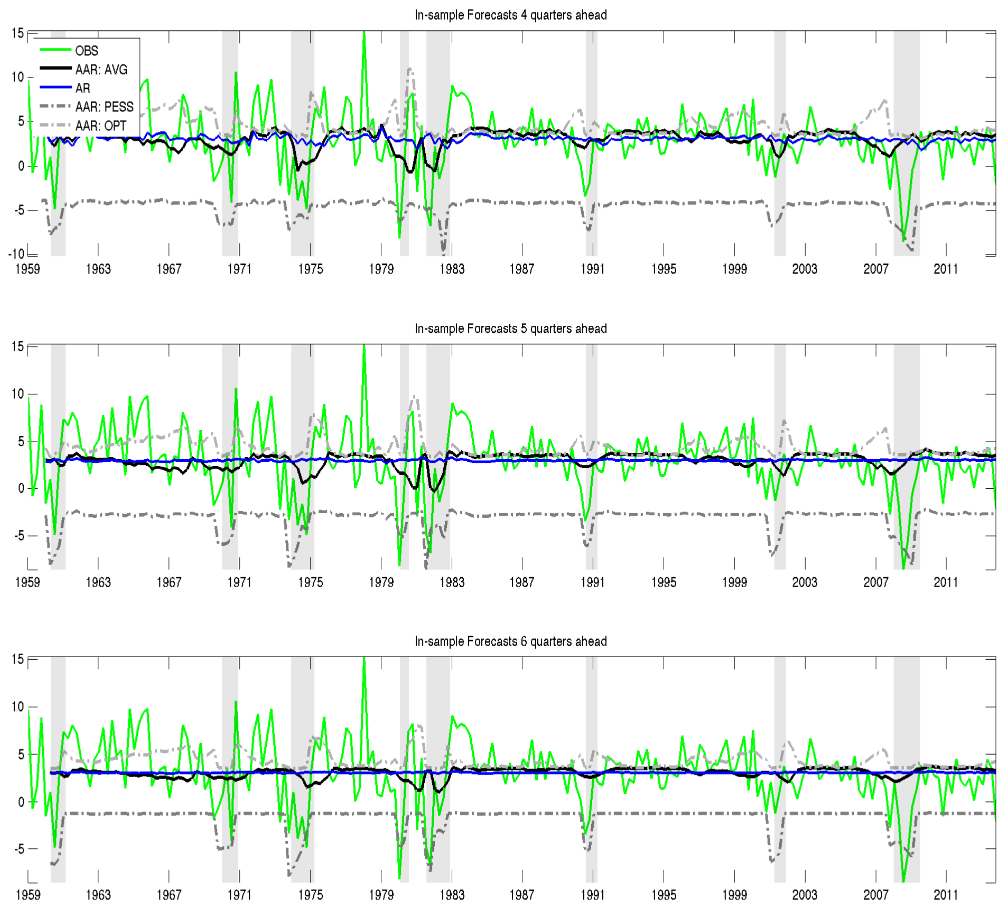
Figure 4: In-sample predictions of GDP growth one to three quarters ahead: AR vs. AAR



This figure shows 1 to 3 quarters ahead in-sample forecasts of GDP growth obtained from the AR and AAR models as well as the optimistic and pessimistic forecast scenarios. Grey areas indicate NBER recessions.

ρ to be constant across states. Second, the in-sample fit of the MS models is better than that of the AAR during certain recessions (e.g., the great recession of 2007-2009) and worse during others recessions. Third, trajectories of fitted values obtained from the MS models are roughly flat during expansion periods as soon as the forecast horizon exceeds $h = 3$. Indeed, the MS models inherit the limitations of the AR model identified on Figure 3 so that the autoregressive coefficients are not significantly different from zero at long horizons.

Figure 5: In-sample predictions of GDP growth four to six quarters ahead: AR vs. AAR



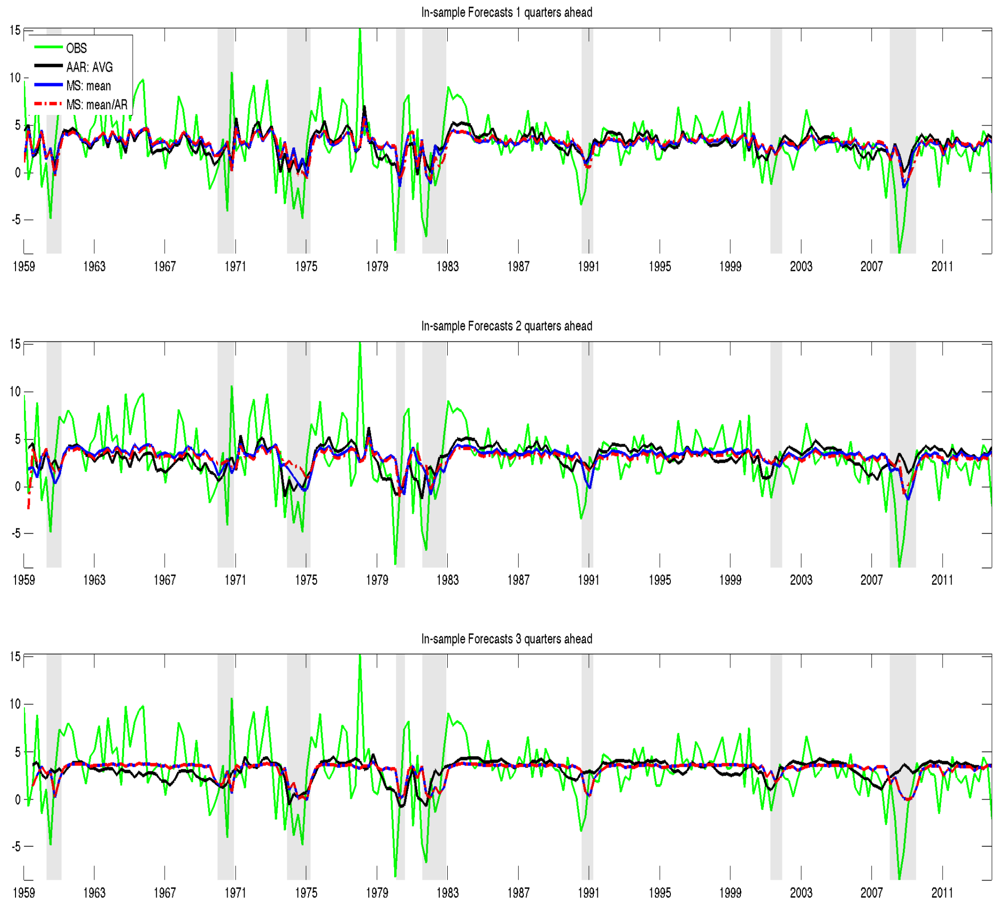
This figure shows 4 to 6 quarters ahead in-sample forecasts of GDP growth obtained from the AR and AAR models as well as the optimistic and pessimistic forecast scenarios. Grey areas indicate NBER recessions.

5.2 Out-of-Sample Analysis of U.S. Recessions

Policy makers and investors attach a high value to forecast accuracy during recession episodes. Typically, recession episodes are shorter than expansion episodes and characterized by rapid changes in the values of many economic indicators. Moreover, economic activity data that are released during recession episodes tend to undergo more or less important revisions. It is therefore of interest to assess how well our models perform during those periods where economic data are harder to track than usual.

In this section, we conduct a real time event study of the last four recessions experienced by the U.S. economy. For each recession, we define a time window that starts around four

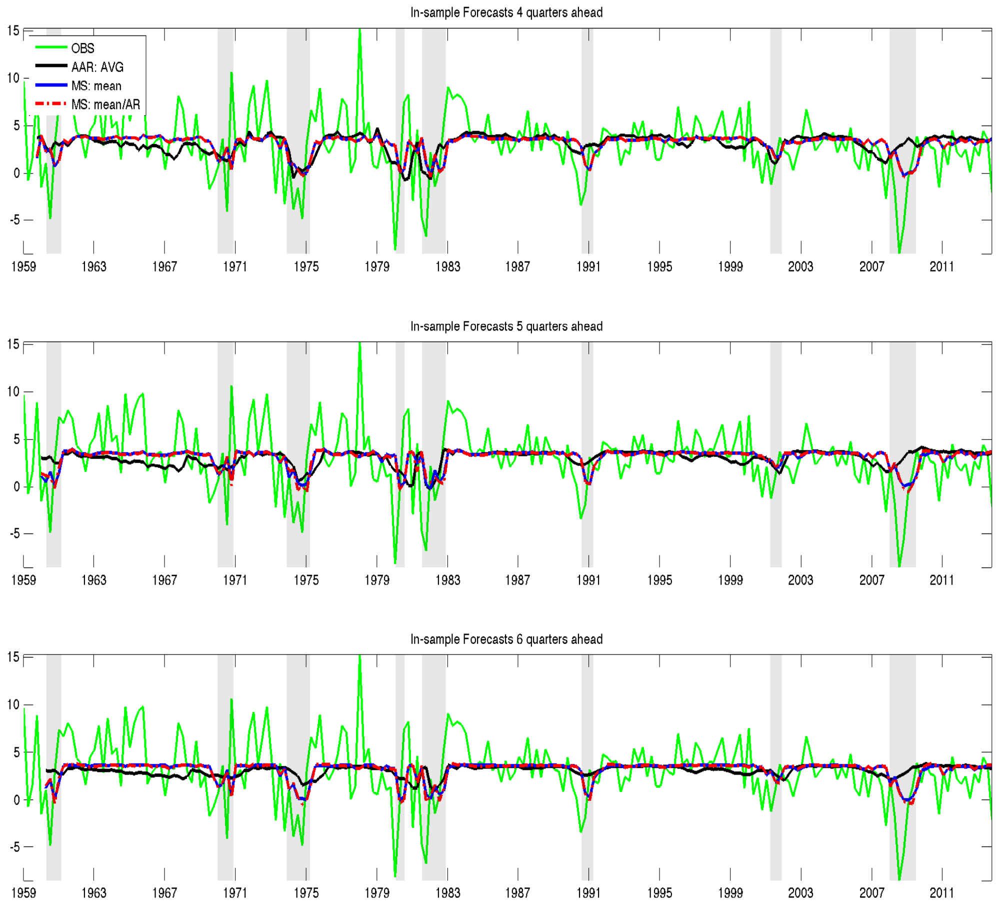
Figure 6: In-sample predictions of GDP growth one to three quarters ahead: AAR vs. MS



This figure compares the trajectories of 1 to 3 quarters ahead in-sample forecasts of GDP growth obtained from the AAR and two MS models: (i) state-dependent intercept, (MS: mean); (ii) state-dependent intercept and autoregressive coefficient (MS: mean/AR).

periods before the recession and ends around two periods after the recession. Considering each recession in turn, we generate the term structure of probabilities of recession for each period on the selected window. Second, we compare the term structure of forecasts of the AAR versus AR and ADL models on two separate figures. Third, we compare the term structure of forecasts of the AAR and MS models on another graph. The MS model considered here is the one with changing intercept and constant autoregressive coefficient. Finally, we generate trajectories of recursive forecasts for the AAR model at fixed horizons. The analysis is done in real time, meaning that the forecasts are generated using the most

Figure 7: In-sample predictions of GDP growth four to six quarters ahead: AAR vs. MS



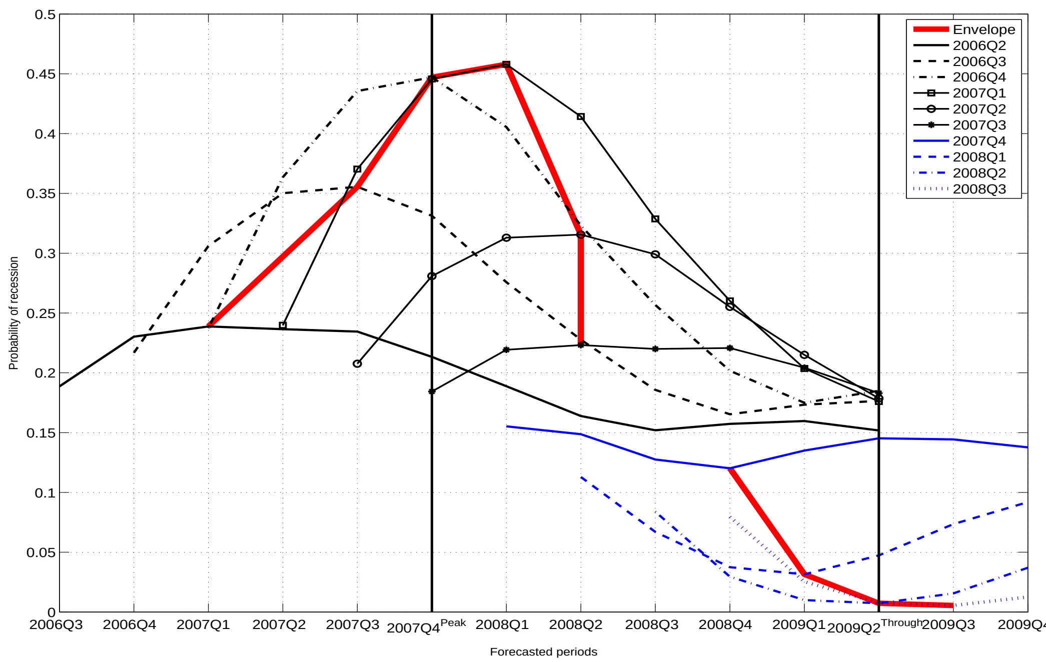
This figure compares the trajectories of 4 to 6 quarters ahead in-sample forecasts of GDP growth obtained from the AAR and two MS models.: (i) state-dependent intercept, (MS: mean); (ii) state-dependent intercept and autoregressive coefficient (MS: mean/AR).

recent GDP release and parameters that are estimated from a sample that stops at the previous official NBER turning point.

5.2.1 The Great Recession (2007-2009)

Figure 8 shows ten curves, each of them representing term structure of the probability of recession at a given date. Two bold red lines are superimposed in order to ease the visualization of the upper envelope of the six concave curves that are at the beginning of the recession and the lower envelope of the four convex curves that are at the end of the recession. The maximum of the upper envelope curve is located at 2008Q1, one quarter only after the date officially designated by the NBER as the peak of the business cycle. Likewise, the minimum of the lower envelope curve is located one quarter after the official end of the recession. This suggests that the upper and lower envelope curves are informative about the business cycle turning points as hypothesized previously.

Figure 8: Forecasting the Great Recession turning points in real time

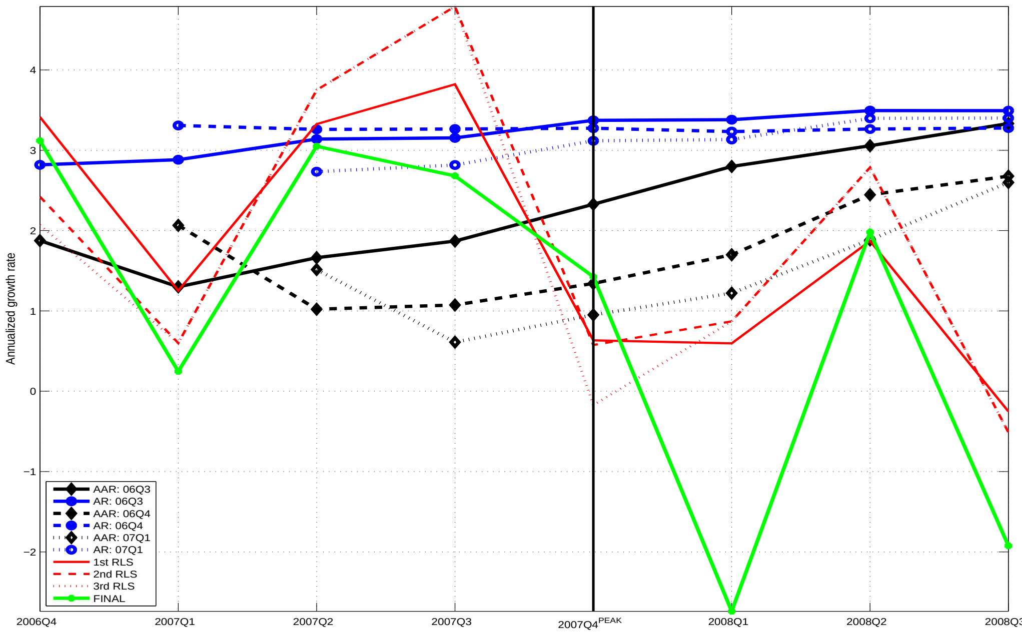


This figure shows out-of-sample predictions of the term structure of recession probabilities obtained from the static Probit model that uses TS only as predictor around the great recession. The black line 2006Q2 corresponds to the forecasts conditional on 2006Q2 information, the black dotted line forecasts is made conditional on 2006Q3, and so on. Thick red lines stand for the upper and lower envelopes of the predicted term structures of recession probabilities.

Figure 9 compares the term structures of out-of-sample predictions for the AAR and AR models. The term structures of out-of-sample forecasts produced by the AR model are roughly flat while those produced by the AAR model exhibit more correlation with the actual data. Clearly, the out-of-sample forecasts of the AR model are more disconnected from reality that those of the AAR model.

Figure 10 compares the term structure of forecasts of the AAR and ADL models. The forecasts of the ADL model are more responsive to the actual data than the those of the AR model, but less responsive than those of the AAR model. The gap between the term structures of forecasts of the AAR and ADL models increases with the horizon.

Figure 9: Direct OOS forecasts of GDP growth during the Great Recession: AR vs. AAR

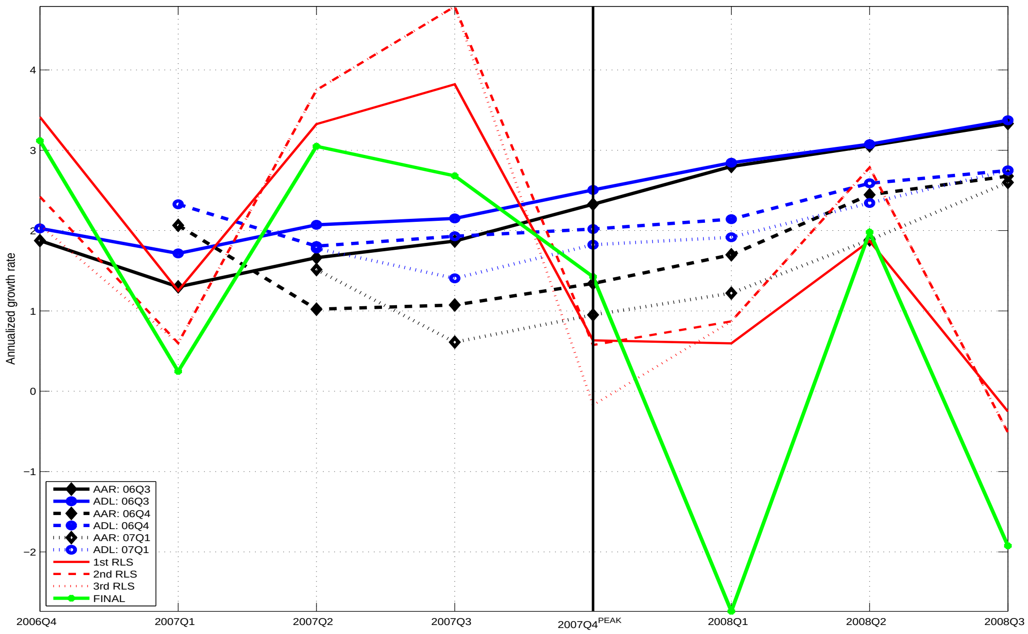


This figure shows out-of-sample forecasts of GDP growth at different horizons obtained from the AR and AAR models around the great recession. For instance, AAR: 06Q3 stands for the forecasts of the AAR model based on the first release of 2006Q3 GDP, as it was available in 2006Q4, and so on. The other lines represent the 1st, 2nd and 3rd releases in real time as obtained from the corresponding vintages. The final data are the most recent values of the 2015Q1 vintage.

Figure 11 compares the term structures of out-of-sample predictions for the AAR and MS models. All term structure of forecasts are flat for the MS model. Their levels a few quarters before and through the middle of the recession are quite deceptive about reality. Indeed, one has to wait until 2009Q1 (one quarter before the official end of the recession) before seeing a sudden drop in the level of the term structure of forecasts of the MS model. Overall, the term structure of out-of-sample forecasts of the MS model has an uninformative shape while its level signals the recession quite late.

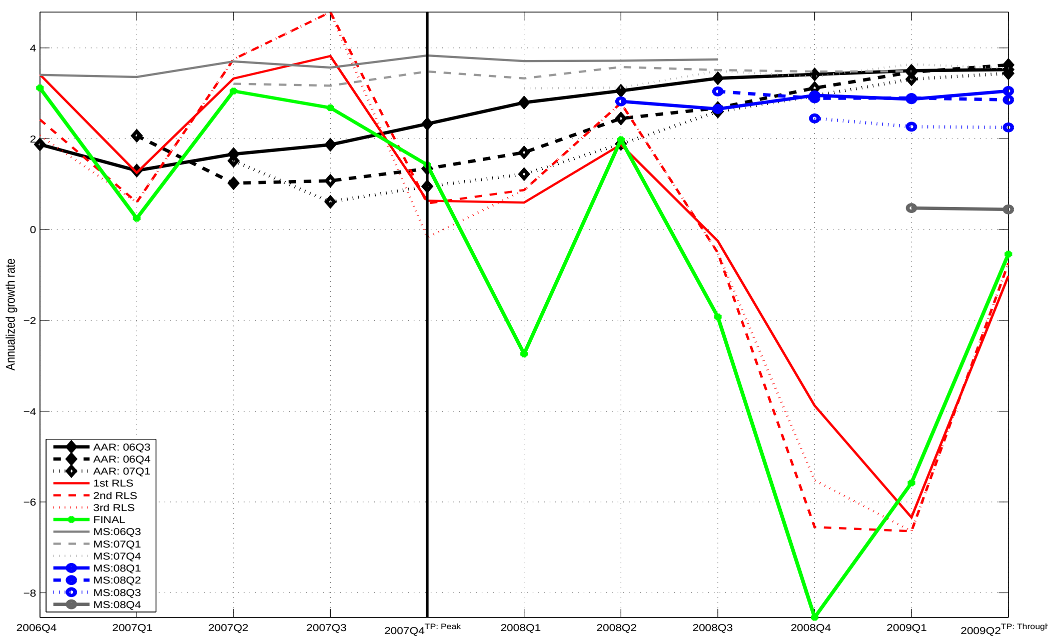
Figure 12 shows the trajectories of recursive forecasts of the AAR model for $h = 3$ and $h = 4$ along with the corresponding optimistic and pessimistic scenarios. The results are quite similar for both horizons. The average forecasts are upward trending during the recession periods while the optimistic and pessimistic forecasts are decreasing. The actual realizations of GDP growth are much lower than the average forecasts. Indeed, actual realizations are closer to the pessimistic forecasts between 2008Q4 and 2009Q1. Having a pessimistic scenario

Figure 10: Direct OOS forecasts of GDP growth during the Great Recession: AAR vs. ADL



This figure shows out-of-sample forecasts of GDP growth at different horizons obtained from the AAR and ADL models around the great recession. For instance, AAR: 06Q3 stands for the forecasts of the AAR model based on the first release of 2006Q3 GDP, as it was available in 2006Q4, and so on. The other lines represent the 1st, 2nd and 3rd releases in real time as obtained from the corresponding vintages. The final data are the most recent values of the 2015Q1 vintage.

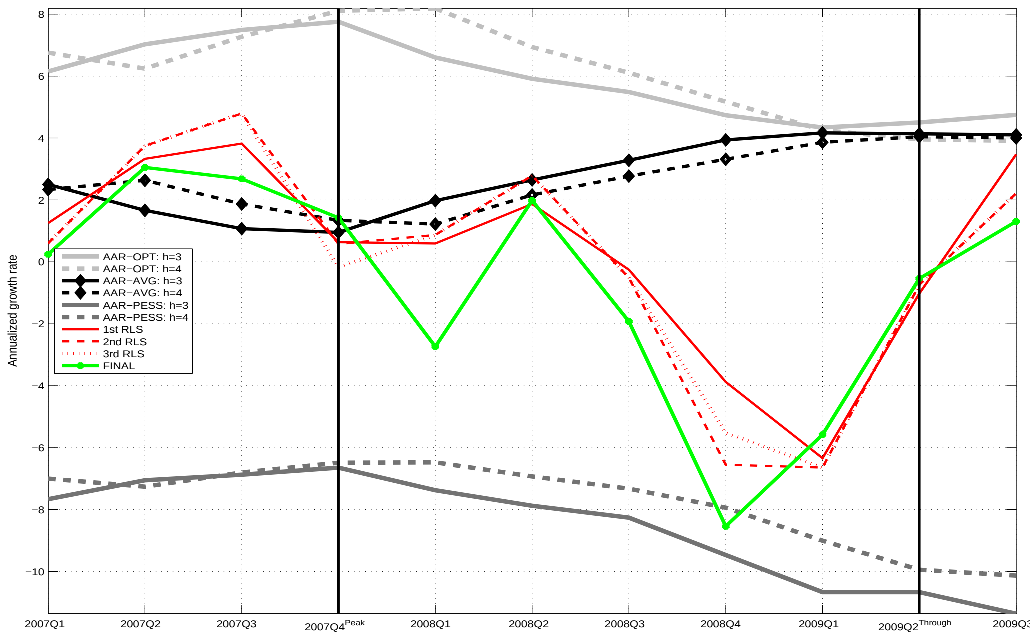
Figure 11: Direct OOS forecasts of GDP growth during the Great Recession: AAR vs. MS



This figure shows direct out-of-sample forecasts of GDP growth at different horizons obtained from the AAR model and the MS model with state-dependent intercept around the great recession.

available in advance can help mitigate the impact of bad surprise during recessions.

Figure 12: Recursive OOS forecasts of GDP growth during the Great Recession: average, optimistic and pessimistic scenarios



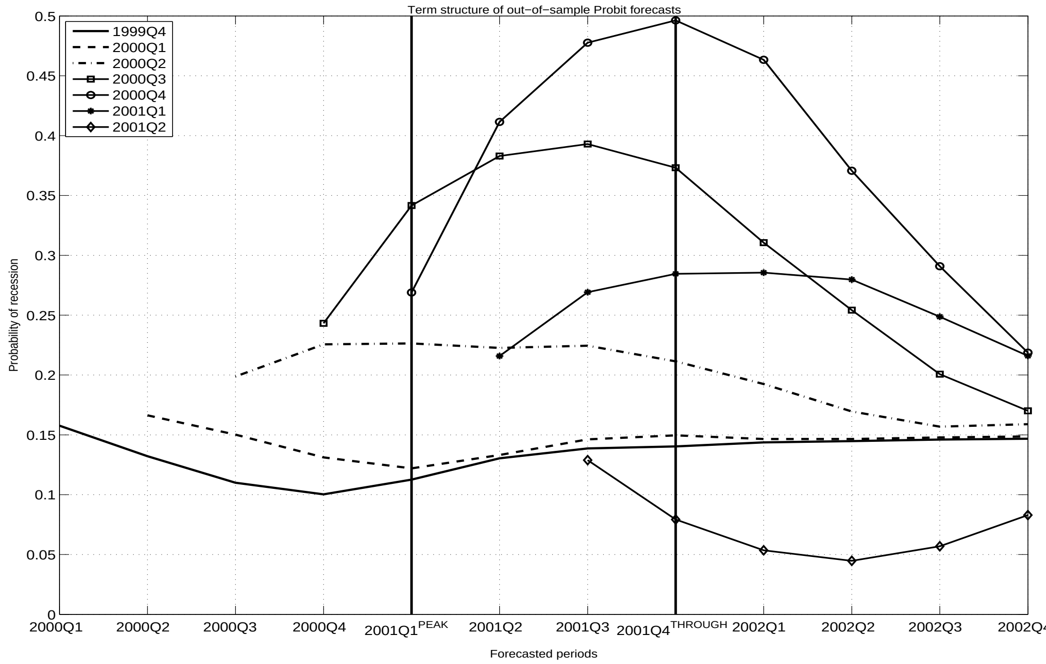
This figure shows the trajectories of recursive forecasts of the AAR model for $h = 3$ and $h = 4$ along with the corresponding optimistic (AAR-OPT) and pessimistic (AAR-PESS) scenarios around the great recession.

5.2.2 The 2001 Recession

The strategy which consists of using the maximum of the upper envelope of the term structure of recession probabilities and the minimum of its lower envelope as turning points does not work well for the 2001 recession (See Figure 13). This might be due to the fact that this recession has been much shorter than the Great recession, or that it has different causes from the other recessions so that it is not predictable by spreads. See discussions in (Ng & Wright 2013), (Kim & Murray 2002) and (French 2005). The beginning and end of this recession are signaled by sudden changes in the shape of the term structure of the probability of recession. Indeed, this curve switched from convex to concave at the beginning of the recession and from concave to convex at the end of the recession. This suggests that changes in the shape of the term structure of recession probabilities signal turning points.[10]

[10]This further suggest that the great recession of 2007 begun at least a year before the official date announced by the NBER. See Figure 8.

Figure 13: Forecasting the 2001 recession turning points in real time



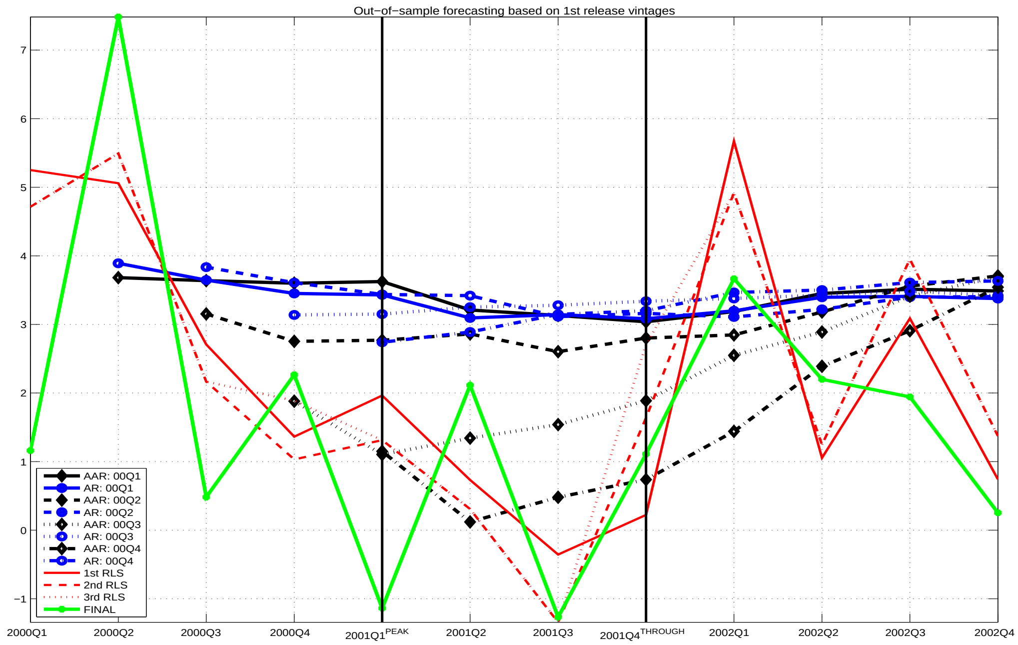
This figure shows out-of-sample predictions of the term structure of recession probabilities obtained from the static Probit model that uses TS only as predictor around the 2001 recession. The black line 1999Q4 corresponds to forecasts conditional on 1999Q4 predictors value, etc..

As previously, the term structure of out-of-sample forecasts of GDP growth generated by the AR model remain quite flat over time (see Figure 14). The term structure of out-of-sample forecasts generated by the AAR model are more informative about the actual economic condition. At glance, we see that the fit of the AAR model is better here than for the Great recession.

Figure 15 shows the term structure of forecasts for the AAR and ADL models. The conclusions are consistent with what was seen previously. That is, the AAR model performs better that the ADL model at glance. As we update the term structure of forecasts, both models fit the data better than during the great recession.

Figure 16 compares the term structure of out-of-sample forecasts for the AAR and MS models. Again, the results are similar to those obtained for the great recession. The shape of the term structure of out-of-sample forecasts of the MS model is uninformative and flat while its level dropped significantly only after the official end of the recession.

Figure 14: Direct OOS forecasts of GDP growth during the 2001 Recession: AR vs. AAR



This figure shows out-of-sample forecasts of GDP growth at different horizons obtained from the AR and AAR models around the 2001 recession. For instance, AAR: 00Q1 stands for the AAR model forecasts using the first release of 2000Q1 GDP, as it was available in 2000Q2, etc.

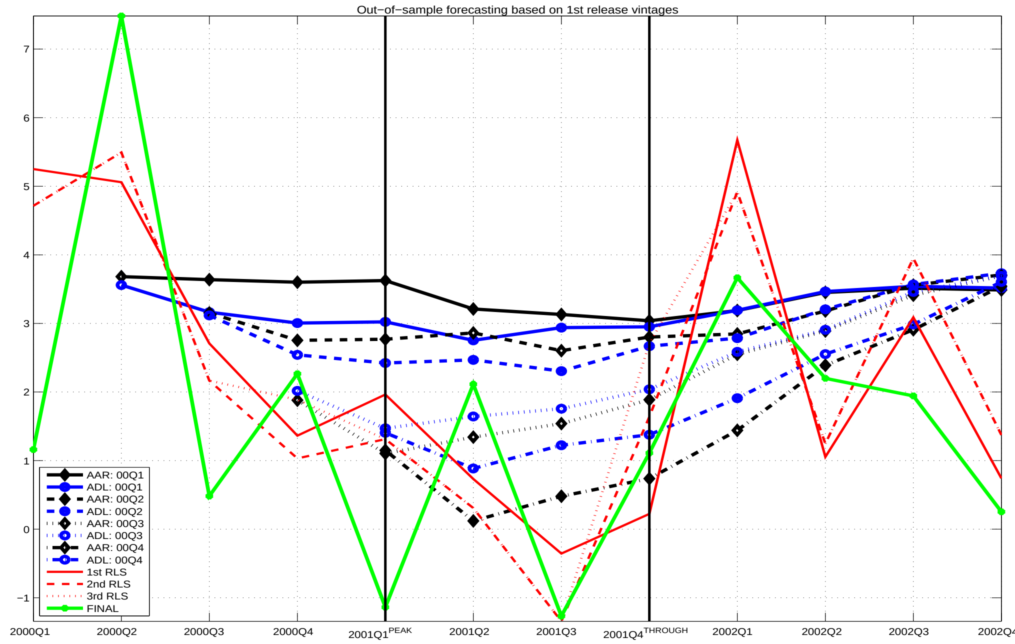
Figure 17 shows the trajectory of recursive forecasts at horizons $h = 3$ and $h = 4$. The actual data are closely tracked by the average forecast. The latter forecast is closer to the optimistic forecast than to the pessimistic forecast, a situation that is probably attributable to the brevity of the 2001 recession.

5.2.3 The 1990-1991 Recession

The recession of 1990-1991 is as short as the one of 2001. However, the dynamic patterns of the term structures of recession probabilities are quite similar to those seen for the great recession of 2007-2009 (see Figure 18). The shapes of these term structure curves may be suggesting that the recession begun at least three quarters before the date officially announced by the NBER and that it lasted until two quarters past the official end date.

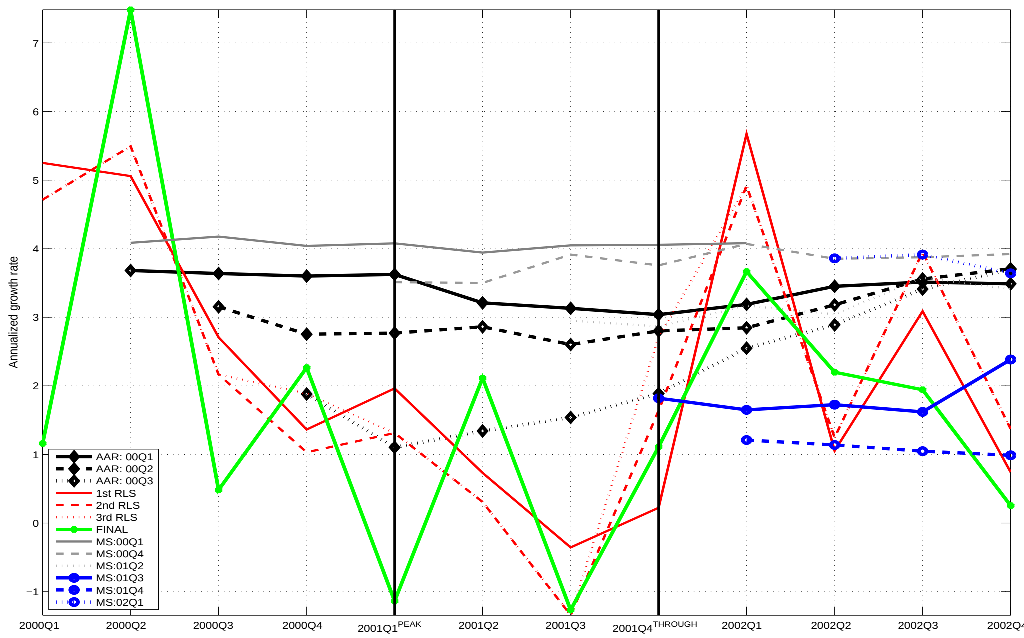
The recession of 1990-1991 was characterized by a sudden drop of GDP growth followed by a quick recovery. It is likely that the brevity of this event makes it hard to predict. Indeed, all term structures of out-of-sample forecasts of GDP growth are flat for the AAR, ADL and AR

Figure 15: Direct OOS forecasts of GDP growth during the 2001 recession: AAR vs. ADL



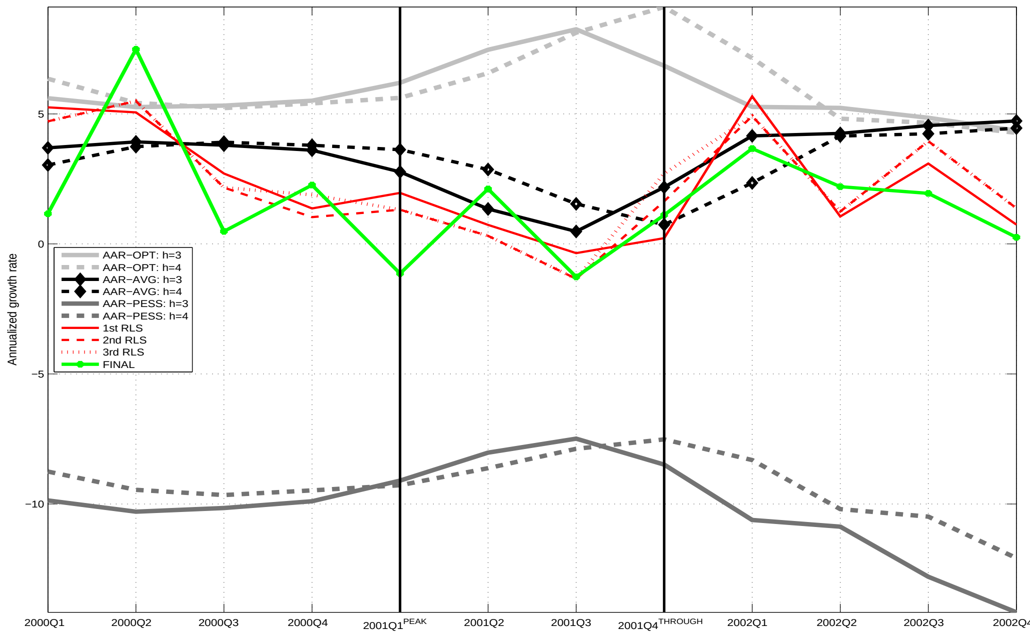
This figure shows out-of-sample forecasts of GDP growth at different horizons obtained from the AAR and ADL models around the 2001 recession. For instance, AAR: 00Q1 stands for the AAR model forecasts using the first release of 2000Q1 GDP, as it was available in 2000Q2, and so on. The other lines represent the 1st, 2nd and 3rd releases in real time as obtained from the corresponding vintages. The final data are the most recent values of the 2015Q1 vintage.

Figure 16: Direct OOS forecasts of GDP growth during the 2001 recession: AAR vs. MS



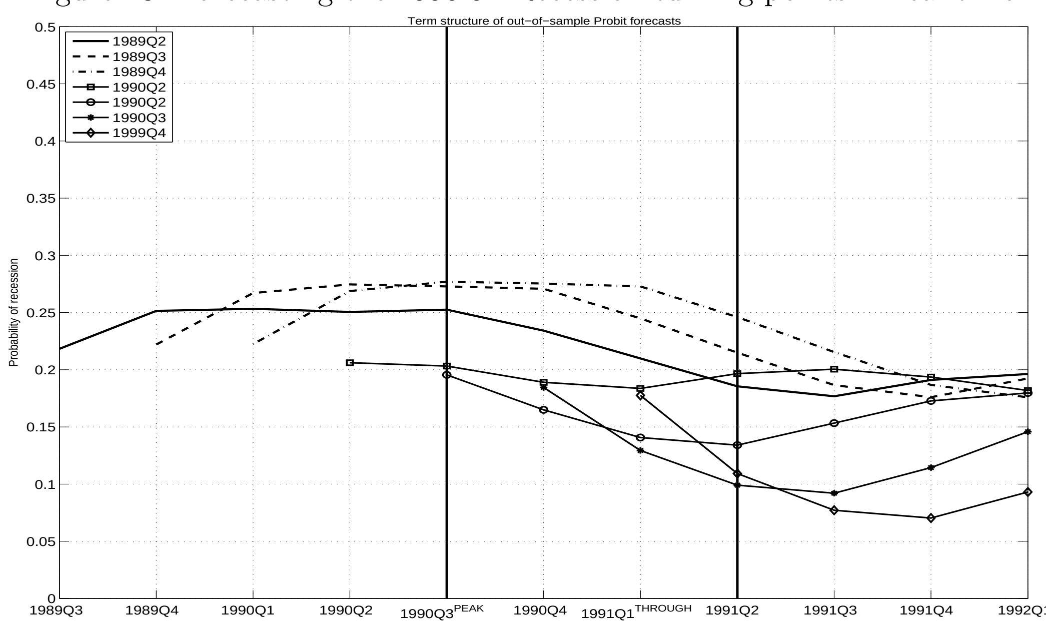
This figure shows direct out-of-sample forecasts of GDP growth at different horizons obtained from the AAR model and the MS model with state-dependent intercept around the 2001 recession.

Figure 17: Recursive OOS forecasts of GDP growth during the 2001 recession: average, optimistic and pessimistic scenarios



This figure shows the trajectories of recursive forecasts of the AAR model for $h = 3$ and $h = 4$ along with the corresponding optimistic (AAR-OPT) and pessimistic (AAR-PESS) scenarios around the 2001 recession.

Figure 18: Forecasting the 1990-91 Recession turning points in real time



This figure shows out-of-sample predictions of the term structure of recession probabilities obtained from the static Probit model that uses TS only as predictor around the 1990-91 recession. The black line 1989Q2 corresponds to forecasts conditional on 1989Q2 predictors value, etc.

models. As discussed in (Ng & Wright 2013), the roots of 1990-91 recession was the savings-and-loan crisis, while the 2001 recession has been initiated by internet bubble. Despite that they both have financial origins, our results confirm those from (Ng & Wright 2013) that neither TS nor CS are able to predict the turning points (see supplementary material). When predicting GDP growth, the AAR model performs well for 2001 recession but has much less predictive power during 1990-91 recession.

As previously, the level of the term structure of out-of-sample forecasts delivered by the MS models signaled the recession with lag (see Figure 21). The fit of the MS model improves significantly after the recession, that is, between 1991Q2 and 1992Q2.

Figure 19: Direct OOS forecasts of GDP growth during the 1990-91 recession: AR vs. AAR

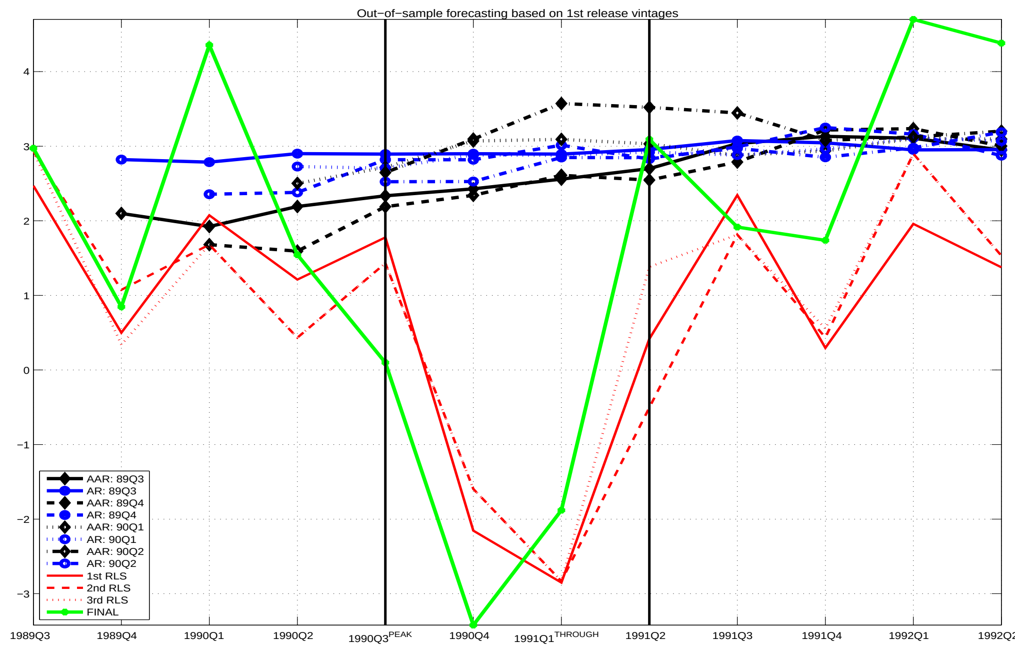
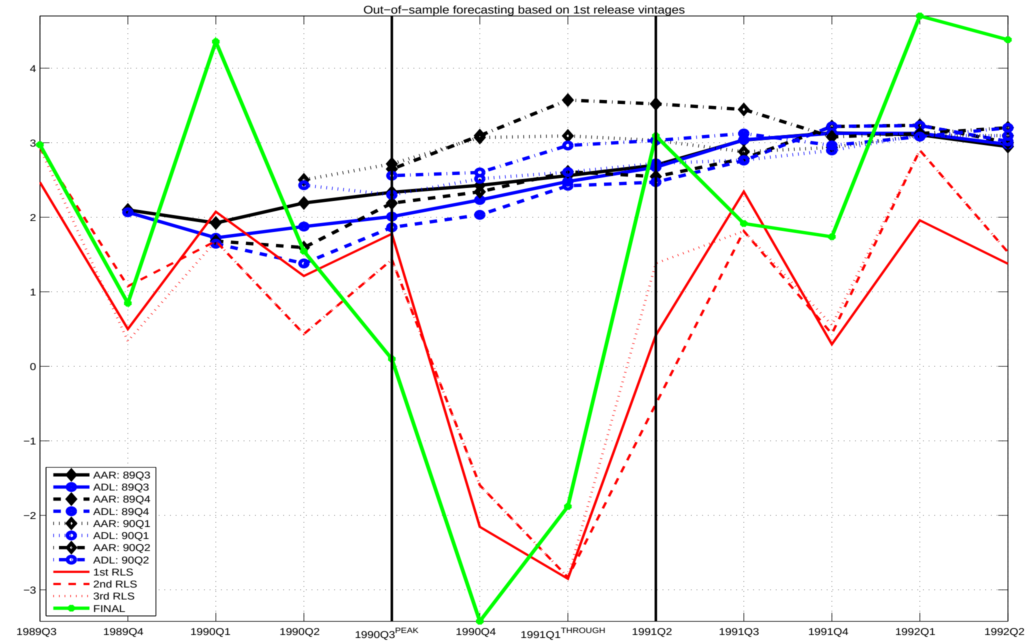


figure shows out-of-sample forecasts of GDP growth at different horizons obtained from the AR and AAR models around the 1990-91 recession. For instance, AAR: 89Q3 stands for the AAR model forecasts using the first release of 1989Q3 GDP, as it was available in 1989Q4, etc.

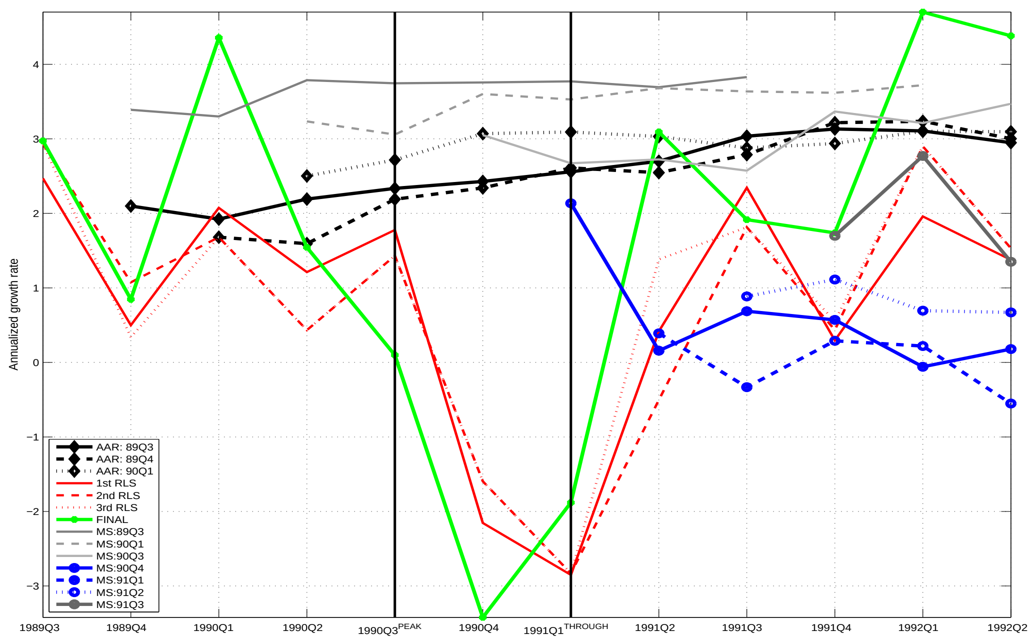
Figure 22 shows the trajectories of the average, optimistic and pessimistic forecasts along with the actual data. The average forecast is quite close to the optimistic forecast and it generally overestimates the actual data.

Figure 20: Direct OOS forecast of GDP growth during the 1990-91 recession: AAR vs. ADL



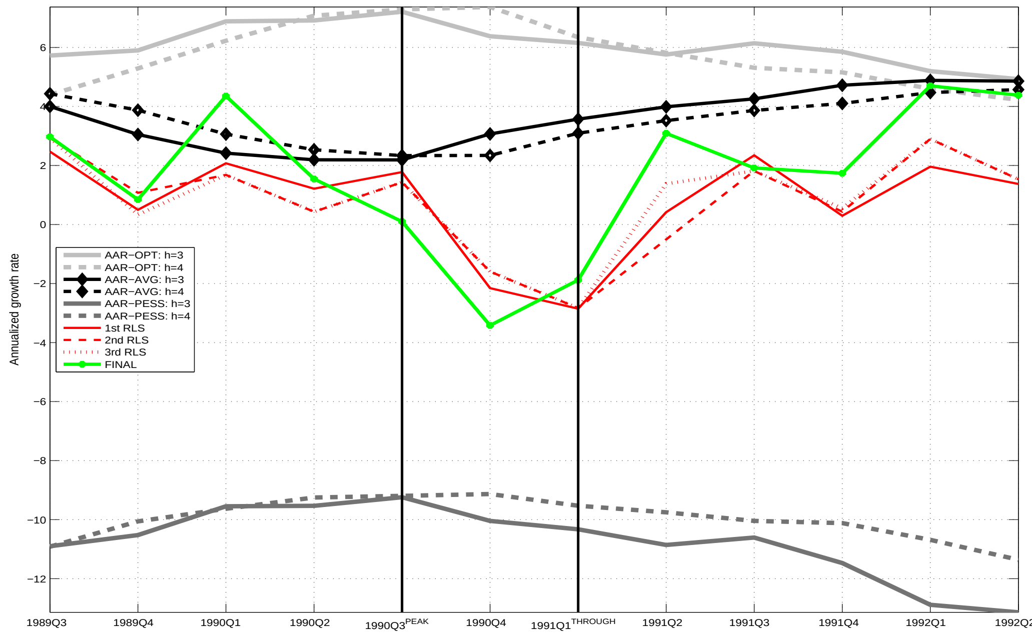
This figure shows out-of-sample forecasts of GDP growth at different horizons obtained from the AAR and ADL models around the 1990-91 recession. For instance, AAR: 89Q3 stands for the AAR model forecasts using the first release of 1989Q3 GDP, as it was available in 1989Q4, and so on. The other lines represent the 1st, 2nd and 3rd releases in real time as obtained from the corresponding vintages. The final data are the most recent values of the 2015Q1 vintage.

Figure 21: Direct OOS forecast of GDP growth during the 1990-91 recession: AAR vs. MS



This figure shows direct out-of-sample forecasts of GDP growth at different horizons obtained from the AAR model and the MS model with state-dependent intercept around the 1990-91 recession.

Figure 22: Recursive OOS forecasts of GDP growth during the 1990-91 recession: average, optimistic and pessimistic scenarios



This figure shows the trajectories of recursive forecasts of the AAR model for $h = 3$ and $h = 4$ along with the corresponding optimistic (AAR-OPT) and pessimistic (AAR-PESS) scenarios around the 1990-91 recession.

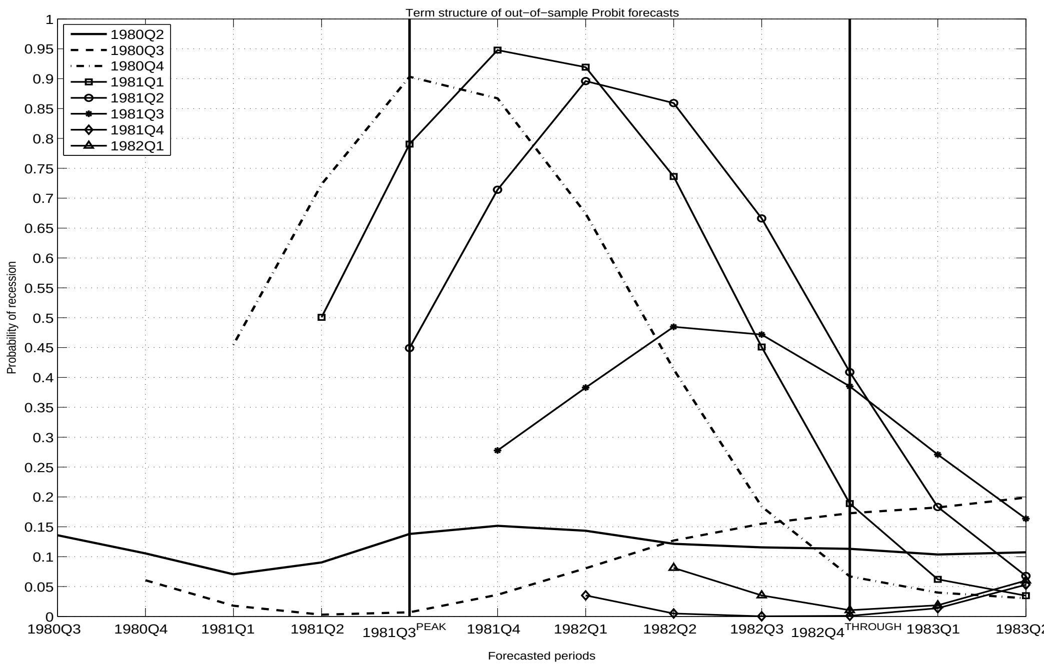
5.2.4 The 1981-1982 Recession

Figure 23 perfectly illustrates how we would like to see the term structure of recession probabilities used to predict turning points. Indeed, the maximum of the upper envelope of the term structures of recession probabilities and the minimum of its lower envelope predict the beginning and end of the 1981-1982 recession quite well. Moreover, the shape of the term structures of recession probabilities changes from convex to concave at the beginning of the recession and from concave to convex at the end of the recession.

The term structure of out-of-sample forecasts delivered by the AAR model fits the actual data quite well during this recession, unlike the forecasts obtained from the AR models which remained flat and uninformative (see Figure 24).

Figure 25 compares the terms structure of forecasts of the AAR and ADL models. We see again that during recessions, it becomes very important to relate the prediction of GDP growth to a nonlinear probability of recession term. This is seen by contrasting the small

Figure 23: Forecasting the 1981-82 recession turning points in real time



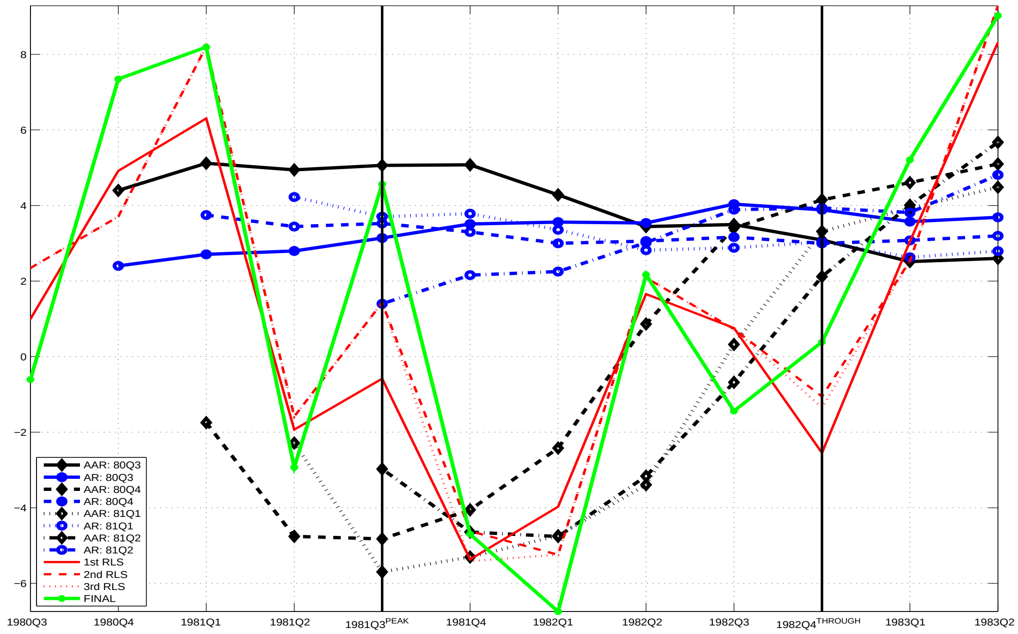
This figure shows out-of-sample predictions of the term structure of recession probabilities obtained from the static Probit model that uses TS only as predictor around the 1981-82 recession. The black line 1980Q2 corresponds to forecasts conditional on 1980Q2 predictors value, etc..

gap between the term structure of forecasts of the two models at 1980Q4 to the big gaps that occur as we move closer to the recession (1981Q1 and 1981Q2).

Three quarters ahead of the beginning of this recession (1980Q4), the out-of-sample forecasts of the MS model were more pessimistic than those of the AAR model (see Figure 26). The superiority of the AAR models is seen by looking at the term structure curves one quarters later (1981Q1). Indeed, the term structure of forecasts associated with the AAR model undergoes a big adjustment while the one associated with the MS model remains quite flat and at a higher level. The out-of-sample forecasts of the MS model fit the data slightly better after the second half than during the first half of the recession.

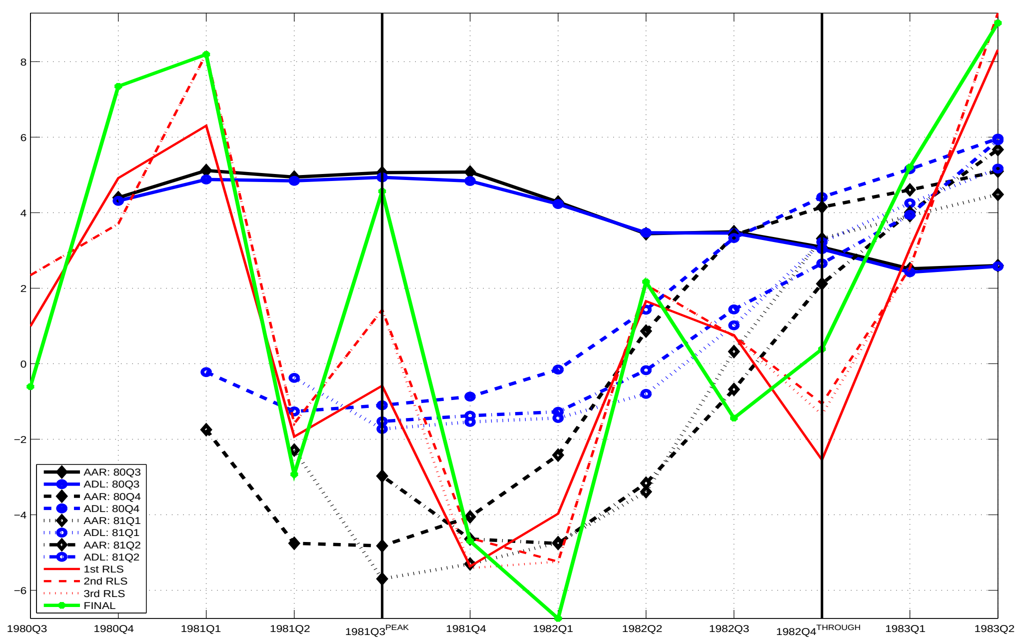
The average out-of-sample forecast delivered by the AAR model fits the data quite well around and during the recession (see Figure 27). The trajectory of the actual data went quite close to the pessimistic forecast at the beginning of the recession. One quarter after the recession, the actual GDP growth went above our optimistic forecast.

Figure 24: Direct OOS forecast of GDP growth during the 1981-82 recession: AR vs. AAR



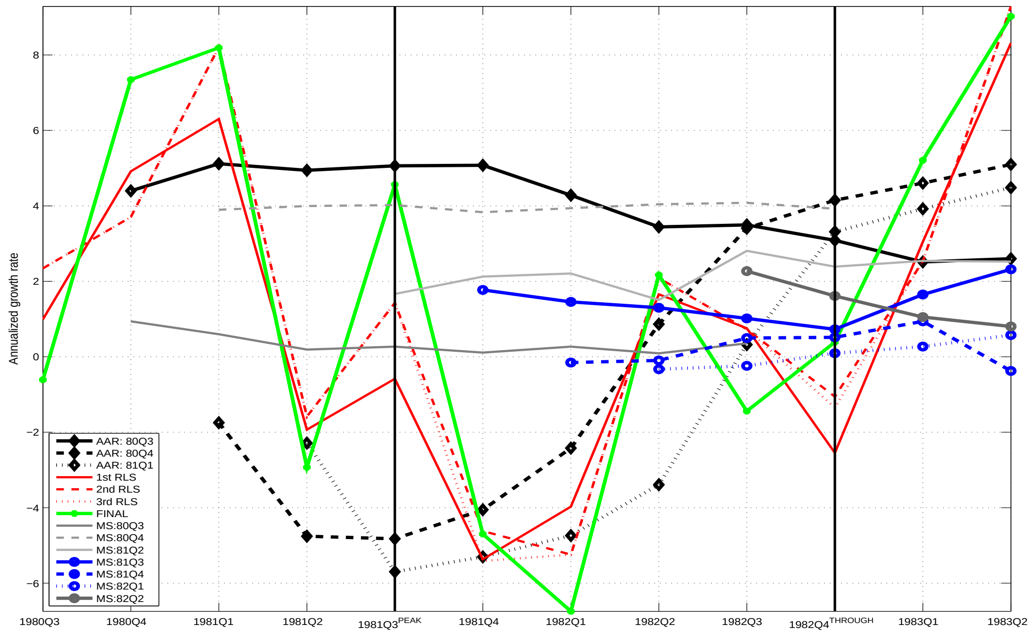
This figure shows out-of-sample forecasts of GDP growth at different horizons obtained from the AR and AAR models around the 1981-82 recession. For instance, AAR: 80Q3 stands for the AAR model forecasts using the first release of 1980Q3 GDP, as it was available in 1980Q4, etc.

Figure 25: Direct OOS forecast of GDP growth during the 1981-82 recession: AAR vs. ADL



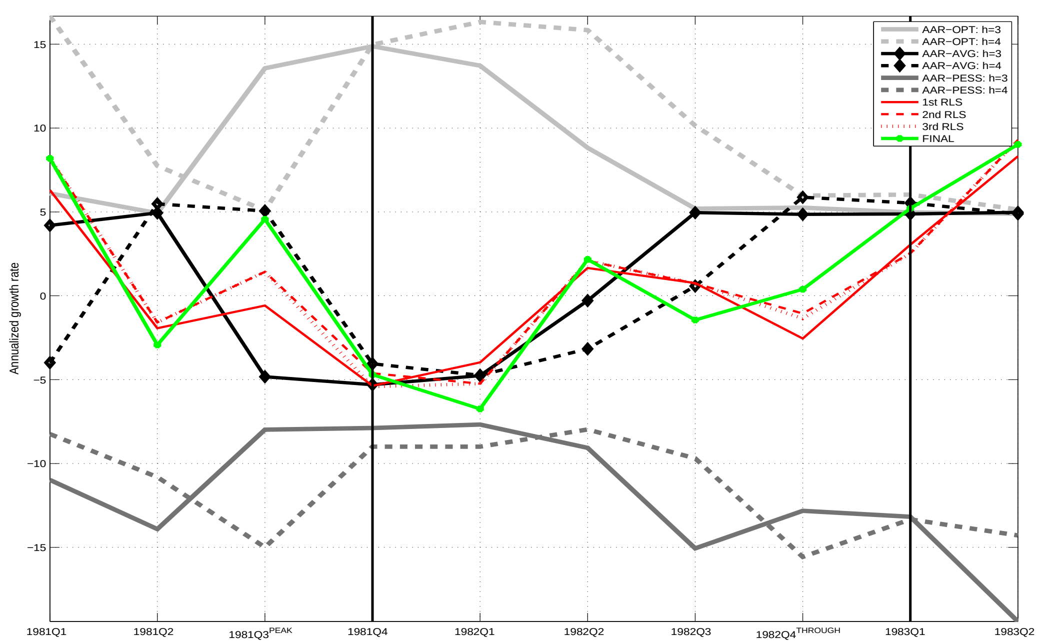
This figure shows out-of-sample forecasts of GDP growth at different horizons obtained from the AAR and ADL models around the 1981-82 recession. For instance, AAR: 80Q3 stands for the AAR model forecasts using the first release of 1980Q3 GDP, as it was available in 1980Q4, and so on.

Figure 26: Direct OOS forecasts of GDP growth during the 1981-82 Recession: AAR vs. MS



This figure shows direct out-of-sample forecasts of GDP growth at different horizons obtained from the AAR model and the MS model with state-dependent intercept around the 1981-82 recession.

Figure 27: Recursive OOS forecast of GDP growth during the 1981-82 Recession: average, optimistic and pessimistic scenarios



This figure shows the trajectories of recursive forecasts of the AAR model for $h = 3$ and $h = 4$ along with the corresponding optimistic (AAR-OPT) and pessimistic (AAR-PESS) scenarios around the 1981-82 recession.

6 Prospects Since the Last Official NBER Turning Point

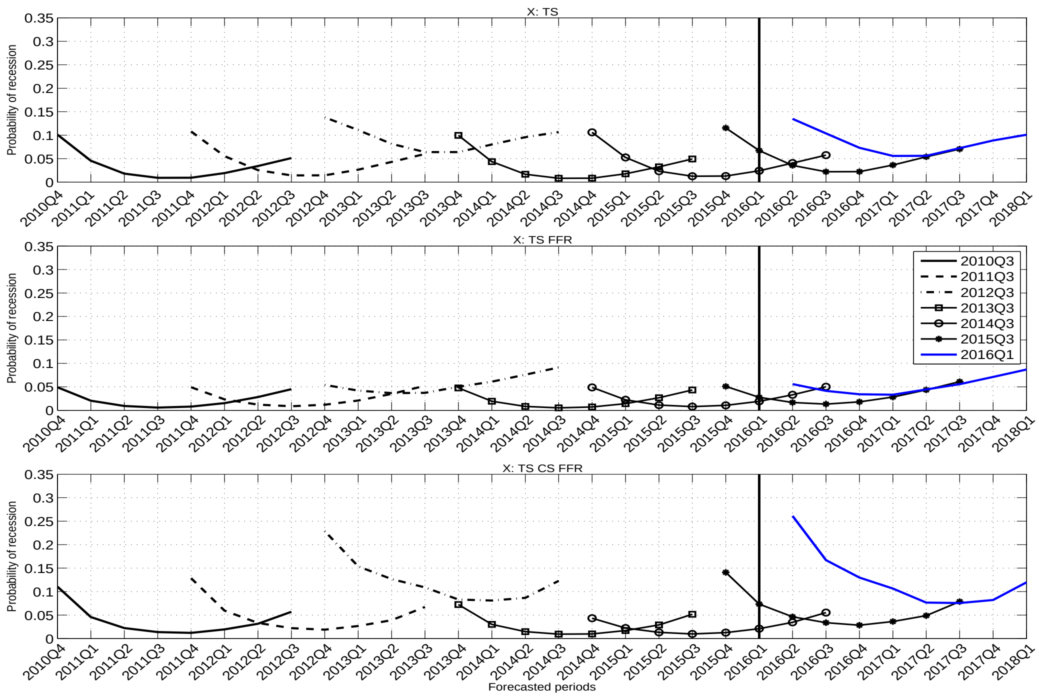
Our previous out-of-sample forecasting exercise focuses mostly on recession episodes and underscores that GDP growth is nonlinearly related to some predictors (such as TS) during critical periods. Our modeling choice leads us to advocate that the relevant nonlinear transformation of the regressors is the probability of a recession at the forecast horizon. Before concluding this paper, we consider backcasting the probability of a recession and GDP growth since the last official turning point and generate predictions for the coming quarters.

We estimate three separate Probit models using [TS], [TS, FFR] and [TS, CS, FFR] and use each model to produce term structure of recession probabilities. Figure 28 shows the estimated sequences of term structures curves since 2010Q4. We see that the three Probit models essentially agree at all dates and horizons. All term structures of recession probabilities are convex, which suggest that there has been no harbinger of a recession since 2010Q4 and that there is none to fear at least until 2018Q1.[11] One might worry that the probabilities of recession predicted by the model based on [TS, CS, FFR] are slightly higher than those predicted by the models based on [TS] and [TS, FFR]. However, we have learned from our empirical experiments that a recession is not imminent as long as the term structure of recession probabilities remain convex.

Figure 28 shows the predictions of GDP growth generated by AAR model for $h = 2$, 3 and 4. The first step Probit model uses [TS, CS, FFR]. The probabilities of recession are estimated to be low at the first step. As a result, the average predictions of the AAR model are quite optimistic. With no surprise, the predictions for $h = 2$ are more informative about the actual data. The bold red line at the right end shows the term structure of out-of-sample forecasts for $h = 1, 2, ..., 8$. These forecasts suggest that the prospects are quite quite good for the US economy. The GDP growth rate is expected to lie around 2.5% in 2016Q3. Going forward, GDP growth is expected to increase steadily and stabilize around 3.5% between 2017Q2 and 2018Q1.

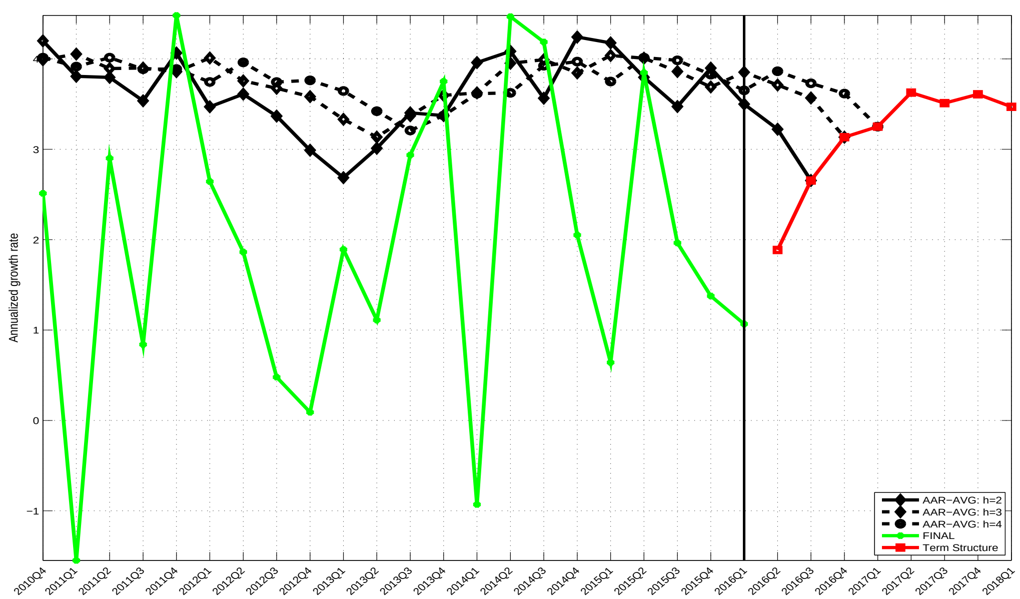
[11] We have investigated models that use stock prices and consumer expectations as additional predictors. The results are quite similar and are presented in the supplementary material.

Figure 28: Real time predictions of recession probabilities since the last NBER announcement



The figure shows out-of-sample predictions of the term structures of recession probabilities obtained from the static Probit model since the last official NBER turning point.

Figure 29: Recursive OOS average forecasts of GDP growth since last NBER announcement



This figure shows the trajectories of recursive forecasts of the AAR model for $h = 2, 3, 4$ along with the term structure of the forecasts conditional on 2016Q1. The Probit model estimated at the first step is conditioned on TS, FFR and CS.

7 Conclusion

This paper explores an approach based on augmented autoregressive models (AAR) to forecast future economic activity. Average forecasts are obtained from an AR(1) model that is augmented with a variable that measures the probability of a recession. AR(1) models augmented with inverse Mills ratios are used to produce forecasts of economic activity conditional on whether the horizon of interest is a recession period (pessimistic forecast) or not (optimistic forecast). The implementation of these models require a prior estimation of a Probit model for the probability of a recession. Overall, our methodology is simple, parsimonious and easy to replicate. It can be easily adapted to other contexts by replacing the economic activity variable by another variable of interest (unemployment rate, credit volume, etc.) and adding more predictors to the first step Probit model. In particular, our pessimistic forecast can be used as input for stress testing scenarios in the banking and real estate sectors.

We find that a static Probit model that uses only the TS as regressor provides comparable in-sample and out-of-sample fit to the data as more sophisticated Probit models. It turns out that the dynamic patterns of the term structure of recession probabilities are informative about business cycle turning points. Indeed, our out-of-sample analyses suggest that the term structure of recession probabilities switches from convex to concave near to the beginning of recessions and from concave to convex near to the end of recessions.

Our most parsimonious AAR model delivers better out-of-sample forecasts of GDP growth than the AR, ADL and Markov Switching models. However, GDP growth is more difficult to predict during certain recessions than during others. As a consequence, the performance of the AAR model is not uniform across recessions. The actual data realizations lie more often above the optimistic forecast than below the pessimistic forecast. We attribute this asymmetry to the fact that the optimistic forecast reflect the combines effect of stagnation and expansion.

Finally, we construct term structures of recession probabilities since 2010Q4 (that is, after the last official turning point). All terms structure curves are convex, which suggest that there has been no harbinger of a recession for the US economy since 2010Q4 and that there is none to fear at least until 2018Q1. The annualized GDP growth is expected to increase steadily from around 2.5% in 2016Q3 to 3.5% in 2018Q1.

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