SEARCH AND MULTIPLE JOBHOLDING

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Search and Multiple Jobholding[*]

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Abstract/Résumé

This paper develops an equilibrium model of the labor market with hours worked, offand on-the-job search, and single as well as multiple jobholders. The model quantitatively accounts for the incidence of and worker flows in and out of multiple jobholding. Central to the model's mechanism is that holding a second job ties the worker to her primary employer, at the benefits of having a stronger outside option to bargain with the outside employer. The model is also informative of how multiple jobholding shapes the outcomes that are typically the focus of search models. Multiple jobholding has opposing effects on job-to-job transitions that mostly offset each other. At the same time, since the option of having second jobs makes the main job survive longer, it reduces job separations and increases the employment rate. These findings have material implications for the calibration of standard models which ignore multiple jobholding.

Cet article développe un modèle d'équilibre du marché du travail avec des heures travaillées, une recherche d'emploi à l'extérieur et à l'intérieur, et des titulaires d'emplois uniques ou multiples. Le modèle rend compte quantitativement de l'incidence du cumul d'emplois et des flux de travailleurs qui y entrent et en sortent. L'élément central du mécanisme du modèle est que le fait d'occuper un second emploi lie le travailleur à son employeur principal, avec l'avantage de disposer d'une option extérieure plus forte pour négocier avec l'employeur extérieur. Le modèle renseigne également sur la manière dont le cumul d'emplois façonne les résultats qui sont généralement au centre des modèles de recherche. Le cumul d'emplois a des effets opposés sur les transitions d'un emploi à l'autre qui se compensent en grande partie. Dans le même temps, puisque la possibilité d'avoir un second emploi permet à l'emploi principal de survivre plus longtemps, elle réduit les cessations d'emploi et augmente le taux d'emploi. Ces résultats ont des implications importantes pour la calibration des modèles standards qui ignorent le cumul d'emplois.

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1   Introduction

The McCall [1970] model, which lays the foundations of the Diamond [1982]-Mortensen [1982]-Pissarides [1985] model, considers the decision problem of an unemployed worker who searches for jobs, randomly receives offers, and rejects them until she finds a suitable job offer. In a large class of models (e.g., Burdett and Mortensen [1998], Postel-Vinay and Robin [2002], Menzio and Shi [2011]), the worker, once in employment, continues to search for jobs and receives offers from outside employers, and rejects them until an offer prompts her to switch employers. A key restriction in these models (and in the search literature in general) is that the worker cannot accept a job at the outside employer while continuing to work at the incumbent employer – that is, she cannot become a multiple jobholder. As a result, nothing is known about the implications of multiple jobholding for the equilibrium properties of these models, and for the inference they provide with regard to labor market dynamics.

In this paper, we develop an equilibrium model of the labor market with single as well as multiple jobholders that addresses these questions. Our motivation is twofold. First, multiple jobholding is a quantitatively important phenomenon, with about one in five workers who work two jobs simultaneously at some point over a 1-year horizon (see Paxson and Sicherman [1996] and Sections 2 and 4 of this paper). Second, search on the job is pervasive (Fallick and Fleischman [2004]; Fujita et al. [2020]), which implies that many workers find themselves at some point contemplating the option to work two jobs simultaneously. In fact, workers who hold only one job have a higher chance of transitioning into multiple jobholding than to make a job-to-job transition. This simple observation begs a further important question, namely what factors push employed workers to switch jobs as opposed to combining two jobs at the same time. Standard search models have nothing to say about this trade-off.

From a modelling perspective, the main challenge facing our analysis is: how to deal with the repeated interaction between one worker and several (two in this paper) employers? This issue is absent from models à la Burdett and Mortensen [1998], since a worker receiving an outside wage offer that beats the current wage moves to the new firm right away. Likewise, in models in the vein of Postel-Vinay and Robin [2002], if the worker holds onto the incumbent employer upon receiving an on-the-job offer, then her wage jumps up and she immediately loses contact with the other employer. This 'no-repeated-interaction' logic is not specific to wage posting or sequential auction models. For instance, in Dey and Flinn [2005] and Cahuc et al. [2006], a worker is at some point in contact with two firms and bargains on the wage, but the sequence is played instantaneously until one of the employers can no longer bid up, i.e. the one-worker-two-employers interaction lasts for an infinitesimal portion of time. In contrast, the model that we develop explicitly allows the worker to be in contact with two employers for possibly many periods. Doing so requires a few key assumptions.

In our model, a worker who receives an on-the-job offer can either reject the offer to stay with the incumbent employer, move to the new employer, or combine the new job with her current job. In the latter event, she cannot quit to the outside employer at a later date – she

already turned down this option by becoming a multiple jobholder –, unless her older job gets (endogenously) dissolved after getting hit by some shock. The worker also cannot search for another job until she gives up the second job, which she can do at any time. Thus, from the perspective of the older employer (henceforth called 'primary employer'), multiple jobholding means a lower risk of losing her worker. In exchange for this commitment, the worker can use the primary job as her outside option to extract a higher surplus from bargaining with the outside (called 'secondary') employer, subject to a participation constraint on the employer's side. This set of assumptions introduces an asymmetry between primary and secondary employers, but this brings two substantial benefits.[1] First, these assumptions make the model tractable. Second, they put strong discipline on worker flows in and out of second jobs, which in turn allows us to assess their relevance by comparing these flows to the data.

The other important feature of the model is hours worked. Workers and firms bargain over both wages and hours, which implies that hours are idiosyncratic to the job match(es) between a worker and her employer(s).[2] Due to the asymmetry between jobs, whether the worker can have a second job depends much on working hours on her primary job: she cannot rebargain these hours to free up some time for the second job. This feature rationalizes the empirical correlation between the hours schedule of jobs and multiple jobholding. To make additional connections to the data, in the model's calibration we propose a mapping of hours worked onto labor market services that creates a meaningful distinction between full-time and part-time employment. This construct is relevant not only to analyze multiple jobholding inflows and outflows, but more broadly it offers a simple solution to capture certain patterns of the intensive margin of labor adjustments (hours per worker) documented in Borowczyk-Martins and Lalé [2019]. Given that these patterns cannot be explained by the recent vintage of search models with fluctuations in hours worked (e.g., Bils et al. [2012], Kudoh and Sasaki [2011], Kudoh et al. [2019], Dossche et al. [2019]),[3] this new construct can be a useful 'add-on' for this class of models.

In sum, the theoretical framework combines a Mortensen and Pissarides [1994]-like model with a structure of very rich adjustments along the intensive margin: in addition to flows in and out of employment, the model features worker movements within the distribution of hours worked, across employers, as well as movements in and out of multiple jobholding. All these variables are determined endogenously. The key notions related to multiple jobholding, such as the primary and secondary jobs, are also endogenous. And since the model is general equilibrium, it is easy to calibrate and usable for counterfactual analysis.

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[1]In some ways, our model configures an environment that is the polar opposite of the Postel-Vinay and Robin [2002, 2004] world, where the worker would use the outside employer to improve working conditions at the incumbent employer. Unlike Postel-Vinay and Robin [2002, 2004], our model is not intended at describing a job ladder. Its aim is to capture spells of second job holding that bring in extra income to the worker and typically last for a short period of time, possibly because of binding participation constraints.

[2]Since hours vary at the job-match level, hours worked change even among *job stayers*. Empirically, these changes are lower than for job changers, but they remain substantial (Borowczyk-Martins and Lalé [2019]).

[3]Borowczyk-Martins and Lalé [2019] show that a large share of cyclical adjustments in hours per worker reflect (within-firm) transitions between full-time and part-time employment, and that they generate sizable and lumpy adjustments in individuals' working hours. Search models with hours fluctuations cannot generate these patterns because they typically feature hours that are the outcome of a smooth optimization problem.

Our analysis yields four main results. First, the model performs well at matching the employment share of multiple jobholders and some salient empirical features of weekly working hours. What is more, it lines up closely to the data with regard to worker flows in and out of second jobs. Given that none of these worker flows are targeted by the calibration, this provides a validation of the model. The model, which is at the monthly frequency, is also consistent with several key observations from annual labor market data. It predicts that about 20 percent of all workers work two jobs simultaneously at some point over a 1-year horizon. The comparisons vis-à-vis the annual data are useful to reveal the role of the underlying heterogeneity in transitions rates in and out of multiple jobs that the model generates. They suggest that the model also does about right along this dimension.

The second main set of results concerns the factors that push or pull workers into second jobs. The model assigns a key role to two parameters in this respect. The first one is on-the-job search efficiency – the rate at which employed job seekers receive offers compared to nonemployed job seekers. Perhaps counterintuitively, on-the-job search efficiency has an ambiguous effect on multiple jobholding. A lower on-the-job search efficiency reduces access to second jobs, but it induces multiple jobholders to hold on longer to their second job given that these jobs become harder to come by. The other key parameter is the flow cost of working a second job, which comes on top of the flow cost of the first job, i.e. the flow value of unemployment in standard search models. The model-implied value of this flow cost amounts to 8 percent of average monthly earnings for men. The corresponding figure for women is 15 percent. For both genders, it is lower than the flow cost of working on the first job, showing that there are returns to scale in working.

Third, introducing multiple jobholding into a standard on-the-job search model pushes down the rate of separation from employment, increases the employment rate, and leads to a slightly higher rate of job-to-job transitions. The main mechanism driving these effects is that the option of having a second job makes workers' main job survive longer. There is a flip side of these results for the calibration of standard search models. These models underestimate the volatility of shocks to match productivity that rationalizes the volume of job separations, i.e. they would need additional volatility to hit the target if they allowed for multiple jobholding; and in order to match the job-to-job transition rate, they require a higher on-the-job search efficiency relative to a world with multiple jobholding. Quantitatively, the differences are important for the volatility of match productivity, and less so for on-the-job search efficiency.

Fourth and last, multiple jobholding matters for the analysis of job creation. To illustrate this point, we use the model to compute the response of job creation to changes in workers' on-the-job search efficiency. We show that there are three channels that mediate this response. A higher on-the-job search efficiency means a larger pool of job seekers; conditional on meeting a job seeker, a higher probability that she is employed as opposed to not having a job; and a higher joint surplus from employment. Multiple jobholding matters mostly for the changing probability of meeting an employed worker. From a firm's perspective, this means a higher risk of becoming a secondary employer, which yields a substantially lower surplus relative to that

of a primary employer. Thus, multiple jobholding dampens the positive feedback from higher on-the-job search efficiency onto additional job creation. Lacking this mechanism, the model would overstate the capacity of on-the-job search to amplify labor market fluctuations.

Our paper contributes to several strands of literature. First and foremost, we substantially expand existing research on multiple jobholding. Much of this literature focuses on understanding the decision to hold a second job from the perspective of the classical labor supply model (Shishko and Rostker [1976], O'Connell [1979], Krishnan [1990]), 'job portfolios' models (Oaxaca and Renna [2006], Hlouskova et al. [2017]), and recently with the added equilibrium effects of education and labor productivity (Auray et al. [2021]). These analyses are conducted in static, frictionless models.[4] Paxson and Sicherman [1996] is an early exception presenting a dynamic model of multiple jobholding. The model, however, is essentially illustrative and not used for quantitative inference. Two recent contributions explore the role of multiple jobholding in the context of partial-equilibrium search models. Mancino and Mullins [2020] analyze the effects of income tax incentives on labor supply (notably the decision to accept second jobs); Lo [2022] studies workers' willingness to work fewer or longer hours given exogenous offers of wage-hour bundles. As far as we are aware, our paper is the first to offer a full-fledged dynamic model of multiple jobholding cast in a general equilibrium setting.[5]

The paper is also related to a long-standing literature in macro and labor economics that studies how and to what extent individuals can adjust their working hours. Altonji and Paxson [1988, 1992] and Blundell et al. [2008], among others, show that workers often need to change job to adjust their hours. Multiple jobholding is relevant in this context because, as Paxson and Sicherman [1996] and Kahn and Lang [1991, 2001] point out, taking on a second job is an effective channel to adjust hours that may offer a valuable alternative to an employer change. Our contribution is to analyze the interplay between adjustments in hours worked, job-to-job transitions and multiple jobholding through the lens of a structural model. To illustrate the relevance of this approach, we show that multiple jobholding affects the behavior of worker flows in and out of employment, which are typically the focus of the search literature. This echoes recent research by Chang et al. [2019] who show that carefully modelling the intensive margin can materially affect our understanding of the behavior of the extensive margin.

The paper is organized as follows. Section 2 describes a few facts about multiple jobholding to help contextualize this analysis. Section 3 presents the model. Section 4 proceeds with the quantification and validation, and Section 5 uses the model to analyze the determinants of multiple jobholding. Section 6 contains the main quantitative results. Section 7 concludes.

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[4]The literature also includes numerous studies that, without relying on a formal theoretical model, offer a wealth of empirical information that is useful for understanding the determinants of multiple jobholding; see Kimmel and Powell [1999], Conway and Kimmel [2001] and Panos et al. [2014].

[5]From a purely formal point of view, the model that is closest to ours is Guler et al. [2012]. They develop a search model of a household formed by a couple which, as the authors point out, is similar to the decision problem of a single agent who would be able to hold two jobs at the same time. They do not push this analogy further, and the main focus of their paper is to understand the reservation-wage behavior of the household under various assumptions on risk preferences and/or job offers being in different locations. Besides this, there are several major differences between Guler et al. [2012] and this paper. *Inter alia*, they have a partial equilibrium model with exogenous wage-offer distributions and they do not consider hours worked.

2 Stylized facts

In this section, we summarize several empirical facts based on Paxson and Sicherman [1996], Conway and Kimmel [1998, 2001], Lalé [2016], Hirsch et al. [2017], and Appendix B of this paper. These facts are useful not only for context purposes but also to motivate and guide the development of our model in the next section.

Before presenting these facts, we must explain how multiple jobholding is defined and measured empirically. According to the U.S. Bureau of Labor Statistics, multiple jobholders are individuals who hold more than one job during some reference period (for instance, the reference week of the Current Population Survey) and who usually receive a wage or salary from the primary job.[6] The vast majority of these workers (over 90 percent) hold only two jobs. A key related definition is that of the primary job. In labor force surveys, this is typically the job with the greatest number of hours worked during the reference period.

2.1 Fact 1: Extent of multiple jobholding in cross-sectional and longitudinal data

When taking a snapshot of the labor market in a given month, about 6 percent of employed workers hold two jobs simultaneously. We call this figure the *multiple jobholding share*. In Lalé [2016], we used data from the Current Population Survey (CPS) to analyze the sources of the dynamics of the multiple jobholding share. The reason why the share is not larger in the cross section is that workers quickly transition out of multiple jobholding (about a 30-percent chance of giving up the second job over a 1-month horizon). At the same time, the probability that a single jobholder in month t becomes a multiple jobholder in $t+1$ is far from trivial. It averages at 2.2 percent, which is the same order of magnitude as the job-to-job transition rate.[7]

Longitudinal data such as the Panel Study of Income Dynamics (PSID) offer complementary information that is useful to appreciate the incidence of multiple jobholding. On average, about 15 percent of individuals work two jobs simultaneously at some point over a 1-year horizon (Paxson and Sicherman [1996]). The figure rises to almost 20 percent when we focus on prime-age workers with some College or higher education (Section 4). Paxson and Sicherman [1996] extend the horizon to the whole working lifetime. They find that the average worker has a 50-percent chance of being a multiple jobholder over the course of her working life.[8]

2.2 Fact 2: Relation between multiple jobholding and part-time / full-time work

The hours schedule of jobs is important to explain multiple jobholding, in the sense that the share of workers working either full-time or part-time in their main job is very different for single and multiple jobholders (Lalé [2016], Kostyshyna and Lalé [2022]). In CPS data, 15 percent of prime-age male multiple jobholders work part-time on their primary job, whereas for those who are single jobholders the chances of working part-time are only 5 percent. For

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[6]Unpaid family workers on their primary job as well as individuals who are self-employed on their primary job and are either self-employed or unpaid family workers on their second job are excluded from this definition.

[7]The job-to-job transition rate of single jobholders is actually lower: it is 'only' 1.6 percent (Table 3).

[8]Figures are for male workers; the number of observations in their sample is too small for female workers.

women, the figures are 33 percent of multiple jobholders working part-time on the primary job vs. less than 20 percent for single jobholders. Perhaps contrary to a popular image, these observations imply that workers who combine two part-time jobs to make a full-time income account for a very small part of multiple jobholding.

2.3   Fact 3: Gradient of multiple jobholding with respect to education

One underappreciated fact about multiple jobholding is its relation with education: multiple jobholding is more, not less, prevalent among more educated workers (Lalé [2019]). The extent of this variation is large. The multiple jobholding share is more than twice higher for workers with a College degree or higher education compared to workers with less than High-school education. Job characteristics could be the underlying reason. More educated individuals are more likely to be in professional and service occupations, which have a higher prevalence of multiple jobholding, possibly due to the greater flexibility of the work schedule afforded by jobs in these occupations. Also of note is that the relation between education and multiple jobholding could be stronger than suggested by the simple unconditional correlation. More educated workers live in larger cities, where a number of factors (long commute times, among others) seem to push the multiple jobholding share down (Hirsch et al. [2017]).

A corollary of these facts is that the side gigs provided by the online platform economy are only partially, if at all, related to the analysis of multiple jobholding presented in this paper. These jobs are more likely to appeal to less educated workers, and if they do, they will also typically be mismeasured by standard labor force survey (Abraham et al. [2017]). We should also note that the correlations summarized in this section have been very stable over time, extending to a period that predates the advent of the online gig economy.

2.4   Fact 4: Relation between the primary and secondary job occupations

Here we describe a new fact about multiple jobholding and provide details in Appendix B. The primary occupation of a multiple jobholder strongly conditions the set of occupations in which she holds her second job.[9] We establish this fact in different ways. We show that for 10-15 percent of all multiple jobholders the occupation of the primary job fully predicts that of the second job, in the sense that 100 percent of multiple jobholders within this primary occupation have the same second job occupation.[10] We can use a less stringent criterion and ask when a single occupation predicts at least 50 percent of the second jobs within the primary occupation of workers. We find that this applies to almost half of all multiple jobholders. Another way to address this question is to analyze instances where two, three, etc. occupations fully predict the second job (again, conditional on workers' primary job occupation). For 54 percent (40 percent) of male (female) multiple jobholders, the answer is: at most 4 second job occupations.

One should not read too much into this fact, but it suggests that the primary and second jobs belong to the same or closely related markets; e.g. a hospitalist provides outpatient

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[9]Paxson and Sicherman [1996] studied the share of multiple jobholders whose second job occupations is the same as that of the primary job. One issue with this statistical measure is that it says nothing about the relation between the two occupations for multiple jobholders who have a different second job occupation.

[10]We define occupations at the 3-digit level. There are 450 occupations at this level of the classification.

services, a teacher conducts a private tutoring business, etc. In this respect, it seems relevant to analyze multiple jobholding in the context of a labor market characterized by a single matching function. It also suggests that, at least in a first approach, one can abstract from permanent job heterogeneity to think about multiple jobholding.

3 A search model with multiple jobholders

3.1 Economic environment

Time $t = 0, 1, \ldots$ is discrete and runs forever. The economy is populated by a unit continuum of workers and by an endogenous measure of employers, both of whom are infinitely lived and discount the future at rate $\beta^{-1} - 1$.

Workers derive utility from market and nonmarket consumptions. They seek to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_t^m + c_t^n \right). \tag{1}$$

Market consumption, c_t^m, consists of labor earnings net of a fixed cost of working ω_j, which is incurred for *each job* that the individual works. The number of jobs held is capped at two, meaning that $j \in \{1, 2\}$. Workers are endowed with one unit of time per period, and h_t denotes hours allocated to market work. Nonmarket consumption, c_t^n, consists of a home-produced good. The production of the home good depends on productivity in the home sector, z_t, which is idiosyncratic to the worker, and on the nonmarket hours of the worker, $1 - h_t$. Specifically, a strictly increasing and concave function $g(.)$ maps nonmarket hours onto home production, such that $c_t^n = z_t g(1 - h_t)$. Home productivity z_t evolves over time according to a persistent stochastic process with transition function G, i.e. $G(z'|z) = \Pr\{z_{t+1} < z'|z_t = z\}$.

The objective of employers is to maximize the expected present value of profit streams π_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t. \tag{2}$$

Each employer has at most one job that is either filled or vacant. A vacant job costs the employer κ units of output per period. A filled job produces a flow of output $y_t f(h_t)$, where y_t denotes idiosyncratic match productivity. $f(\cdot)$ is the function that maps a worker's hours worked at the current employer, h_t, onto labor services. Match productivity y_t is stochastic and exhibits persistence over time. Its transition function is denoted as F. Employers enter the labor market until the value of holding a vacant job equals zero.

Workers and employers come together via search. The number of contacts per unit of time depends on the number of vacancies and number of job seekers. This relationship is governed by a constant-returns-to-scale function, meaning that the job-filling probability, $q(.)$, depends only on labor market tightness θ_t, i.e. the ratio between vacancies and job seekers. Both nonemployed workers and single jobholders search for jobs. On the other hand, if a worker

has two jobs, she must give up either at least one of them to start receiving job offers. The probabilities that a nonemployed worker and that a single jobholder meet an employer with a vacant job are, respectively,

$$\lambda_{0,t} = \theta_t q(\theta_t) \quad \text{and} \quad \lambda_{1,t} = s_e \lambda_{0,t}. \tag{3}$$

In (3), s_e measures on-the-job search efficiency, which is a key parameter of virtually any on-the-job search model. On meeting, match productivity y_t is sampled from a distribution denoted as F_0. After observing the initial y_t, the worker and the employer either walk away from each other or choose to stay together.

We need certain assumptions to accommodate the option to hold two jobs and operationalize key notions related to multiple jobholding. On receiving an outside job offer, the worker can either turn down the offer, move to the new employer (job-to-job transition), or combine the new job with her current job (*multiple jobholding*). In the latter event:

(A1) The worker can give up the second job at any time, but until she does so, she cannot quit her older employer – called *primary employer* –, unless their match gets dissolved.

(A2) Her wages and hours at the primary employer are pinned down by the same bargain as a single jobholder who would have the same match and idiosyncratic home productivity.

(A3) The worker can use the primary job as her outside option to bargain on wages and hours with the new employer – henceforth called the *secondary employer*.

Obviously these assumptions are restrictive along some dimensions, but as we will point out in the next sections, they provide a substantial benefit in terms of tractability. It is also clear that these assumptions have bearing on the predictions of the quantitative model concerning turnover in and out of second jobs. We will show in Section 4 that these predictions come remarkably close to the data, even though the calibration targets none of the inflows and outflows of second jobs. This is clear evidence that Assumptions (A1)–(A3) are empirically relevant to our understanding of multiple jobholding.

Let us make a few additional remarks to complement the assumptions. First, we have mentioned worker-firm bargaining in (A2)–(A3). The bargaining protocol that will be set up in the next sections is such that participation constraints are satisfied on both employers' sides. Second, we add the timing convention that if the worker opts for a job-to-job transition, then she immediately loses contact with the former employer.[11] As a result, the worker cannot use her previous employment status to bargain for a higher wage at the new employer. Vice versa, if the worker discards the option of moving to the outside employer, then this contact is lost immediately and her bargaining position remains unchanged at the current employer.[12] Third, we exclude the possibility for the secondary employer of inducing a quit by paying a wage

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[11]See Pissarides [1994] or more recently Fujita and Ramey [2012] for a similar timing convention.

[12]Note that this together with Assumption (A2) implies that meeting an outside employer leaves a worker's wage and hours worked on the primary job unchanged.

bonus upon meeting the worker. This arrangement is also ruled out in any future periods as the secondary employer must recognize that the worker is committed to her primary employer.

3.2 Asset values and joint match surpluses

From this point on, we drop the time subscript t and use a recursive formulation of workers' and firms' decision problems. We denote by $N(z)$, $E(y_1, z)$ and $E(y_1, y_2, z)$ the asset values of, respectively, nonemployed workers, single jobholders, and multiple jobholders. For firms, we use $J(y_1, z)$ to denote the asset value of an employer matched to a single jobholder. For those matched to a multiple jobholder, we denote by $J_1(y_1, y_2, z)$ and $J_2(y_1, y_2, z)$ the asset values of the primary and secondary employers, respectively. The asset value of holding a vacancy is always zero.

There are two joint match surpluses that we need to define. The first one is the joint match surplus of employment with a single jobholder, $S(y_1, z)$, which is also the surplus of the primary job held by a single jobholder following Assumption (A2). The surplus is given by

$$S(y_1, z) = J(y_1, z) + E(y_1, z) - N(z). \tag{4}$$

Following Assumption (A3), the surplus of the second job of a multiple jobholder, denoted as $S(y_1, y_2, z)$, is defined by

$$S(y_1, y_2, z) = J_2(y_1, y_2, z) + E(y_1, y_2, z) - E(y_1, z). \tag{5}$$

That is, the surplus of multiple jobholding consists of the surplus of the secondary employer and the worker's asset value of holding two jobs net of the value of working only on her primary job. Note that $J_1(y_1, y_2, z)$ does not appear in the system of equations (4) and (5). $J_1(y_1, y_2, z)$ will show up only in the continuation value of $J(y_1, z)$, and thus in the joint surplus $S(y_1, z)$.

3.3 Bargaining

As already mentioned, workers and employers bargain on wages and hours period by period to split the surplus.[13] Letting $\phi \in (0, 1)$ denote workers' Nash bargaining share, they maximize

$$\max_{w, h} \left\{ (E(y_1, z) - N(z))^\phi J(y_1, z)^{1-\phi} \right\}. \tag{6}$$

The solution of (6) yields wages and hours for single jobholders, as well as for the primary job of multiple jobholders under Assumption (A2). Note that since $J_1(y_1, y_2, z)$ is included in the continuation value of $J(y_1, z)$, the wage schedule that comes out of (6) accounts for the fact that a worker may eventually become a multiple jobholder and that this would not change her bargaining position vis-à-vis the primary employer.

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[13]Given that single jobholders and firms rebargain every period, there is no role for wages to reduce worker quitting; see Shimer [2006] for additional discussion of this issue.

Next, following Assumption (A3), wages and hours worked for the second job of a multiple jobholder are pinned down by:

$$\max_{w,h} \left\{ (E(y_1, y_2, z) - E(y_1, z))^{\phi} J_2(y_1, y_2, z)^{1-\phi} \right\}, \tag{7}$$

subject to $J(y_1, z) \geq 0$, i.e. the worker cannot retain the primary job if this job is not viable. For reasons discussed in Section 3.4, it is also the case in equilibrium that $J_1(y_1, y_2, z) \geq J(y_1, z)$, so that this employer is not worse off when her worker chooses to hold a second job. The solution of (7) also ensures participation of the secondary employer, as explained in Proposition 1 below:

Proposition 1. *Wages in primary and second jobs split the surplus in proportion to the agents' bargaining weights:*

$$E(y_1, z) - N(z) = \phi S(y_1, z) \quad and \quad J(y_1, z) = (1 - \phi) S(y_1, z), \tag{8}$$

$$E(y_1, y_2, z) - E(y_1, z) = \phi S(y_1, y_2, z) \quad and \quad J_2(y_1, y_2, z) = (1 - \phi) S(y_1, y_2, z). \tag{9}$$

Since $E(y_1, y_2, z) - E(y_1, z) \geq 0 \Rightarrow J_2(y_1, y_2, z) \geq 0$, the participation constraint of the secondary employer is satisfied.

If the functions $f(.)$ and $g(.)$ are differentiable, then at an interior solution, hours worked in primary and second jobs satisfy, respectively,

$$y_1 f'(h(y_1, z)) = zg'(1 - h(y_1, z)), \tag{10}$$

$$y_2 f'(h(y_1, y_2, z)) = zg'(1 - h(y_1, z) - h(y_1, y_2, z)), \tag{11}$$

i.e. hours worked equalize the marginal product in the market and in the home sector.

Proof. See Appendix A.1. □

Note that the Proposition discusses $J_2(y_1, y_2, z) \geq 0$, but it does not compare it to the surplus $J(y_2, z)$ that the secondary employer would get if the worker would not have a primary job. It is intuitive that $J(y_2, z)$ is higher than $J_2(y_1, y_2, z)$ for most y_2 and z, since the worker has a better outside option compared to that of a single jobholder.[14] Besides this, a second job generates little output, given the immediate corollary of Equations (10)-(11):

Corollary. *Ceteris paribus, a multiple jobholder works fewer hours on her second job compared to a single jobholder employed in that job, i.e.*

$$h(y_1, y_2, z) \leq h(y_2, z) \tag{12}$$

for two workers with the same idiosyncratic home component z and match productivity y_2, regardless of match productivity y_1 of the multiple jobholder in her primary job.

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[14]Thus, in most instances the secondary employer would be willing to pay a fee to the worker to induce her to quit her primary job, but assumption (A1) rules out this type of side payments.

The corollary follows directly from the observation that hours worked in primary job reduce the marginal productivity in the home sector – equivalently, that hours worked in the second job come in at a higher marginal disutility of work.

A corollary to this corollary is that a multiple jobholder is likely to work more hours on her primary job than in her second job. Unlike in the corollary above, we now consider hours of the *same* individual, so that z is fixed. If $y_2 \approx y_1$, then $h(y_1, z) \approx h(y_2, z) \geq h(y_1, y_2, z)$ following Equation (12).[15] $y_2 \approx y_1$ is a reasonable approximation for multiple jobholders because on the one hand if y_2 were much higher than y_1 then the worker would have quit to the outside employer upon meeting; and on the other hand, if y_2 is much lower than y_1, then the second job generates too little surplus to cover the cost of working a second job.

3.4 Bellman equations[16]

We need to first define some policy functions to write the Bellman equations. Proposition 2 below enables us to focus on three functions that correspond to the following binary decisions: (i) an employer's decision to keep a job alive, $p(y_1, z) = \mathbb{1}\{J(y_1, z) \geq 0\}$; (ii) a worker's decision to take on a second job, $d(y_1, y_2, z) = \mathbb{1}\{E(y_1, y_2, z) \geq E(y_1, z)\}$; (iii) a worker's decision to leave the current job upon meeting an incumbent employer, $\ell(y_1, y_2, z) = \mathbb{1}\{\max\{E(y_2, z), N(z)\} \geq \max\{E(y_1, z), E(y_1, z) + p(y_1, z)(E(y_1, y_2, z) - E(y_1, z)), N(z)\}\}$. Observe that inside the 'max' operator $p(y_1, z)$ multiplies $E(y_1, y_2, z) - E(y_1, z)$ as per assumption (A1): the option of having a second job is contingent on the first job being viable.

Proposition 2. *The policy functions $p(y_1, z)$, $d(y_1, y_2, z)$, $\ell(y_1, y_2, z)$ can be expressed jointly as functions of the joint match surpluses $S(y_1, z)$ and $S(y_1, y_2, z)$. Specifically,*

$$p(y_1, z) = \mathbb{1}\{S(y_1, z) \geq 0\} \tag{13}$$

$$d(y_1, y_2, z) = \mathbb{1}\{S(y_1, y_2, z) \geq 0\} \tag{14}$$

$$\ell(y_1, y_2, z) = \mathbb{1}\{p(y_2, z)S(y_2, z) \geq p(y_1, z)(S(y_1, z) + d(y_1, y_2, z)S(y_1, y_2, z))\} \tag{15}$$

Proof. See Appendix A.2. □

With these policy functions at hand, we can describe the Bellman equations of the joint surpluses, $S(y_1, z)$, $S(y_1, y_2, z)$, and the asset value of a primary employer, $J_1(y_1, y_2, z)$. To simplify notations, we also include $N(z)$, the value of being nonemployed, in the system of equations below. $N(z)$ solves

$$N(z) = \beta \int \left(N(z') + \lambda_0 \phi \int p(y_1', z')S(y_1', z')\,dF_0(y_1')\right) dG(z'|z). \tag{16}$$

---

[15]This implication of the model is fully consistent with the empirical definition of the primary job, which is defined as the job that has the greatest number of hours worked; see Section 2.

[16]In this section, we focus on the Bellman equations for $S(y_1, z)$, $S(y_1, y_2, z)$, and $J_1(y_1 y_2, z)$, because these asset values are sufficient to describe the equilibrium of the model. $S(y_1, z)$ and $S(y_1, y_2, z)$ are derived from the Bellman equations that define $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$ through calculations presented in Appendix A.3.

The continuation value of nonemployment includes the surplus of becoming a single jobholder multiplied by the worker's bargaining power.

The joint surplus of employment with a single jobholder is

$$S(y_1, z) = y_1 f(h(y_1, z)) + zg(1 - h(y_1, z)) - (N(z) + \omega_1) + \beta \Bigg(S_e^+(y_1, z) + S_j^+(y_1, z)$$
$$+ \int \left(\int p(y_1', z') \left(1 - \lambda_1 \int \ell(y_1', y_2', z') dF_0(y_2') \right) S(y_1', z') \right) dF(y_1'|y_1) \Bigg) dG(z'|z) \Bigg) \quad (17)$$

where

$$S_e^+(y_1, z) = \int \Bigg(N(z') + \phi \lambda_1 \int \int (\ell(y_1', y_2', z') p(y_2', z') S(y_2', z') + (1 - \ell(y_1', y_2', z'))$$
$$\times p(y_1', z') d(y_1', y_2', z') S(y_1', y_2', z')) dF_0(y_2') dF(y_1'|y_1) \Bigg) dG(z'|z) \quad (18)$$

and

$$S_j^+(y_1, z) = \lambda_1 \int \int \int ((1 - \ell(y_1', y_2', z')) p(y_1', z') d(y_1', y_2', z') (J_1(y_1', y_2', z')$$
$$- (1 - \phi) S(y_1', z'))) dF_0(y_2') dF(y_1'|y_1) dG(z'|z). \quad (19)$$

There are three components in the continuation value of a match with a single jobholder. The first one is the worker's component $S_e^+(y_1, z)$ defined in Equation (18), which captures the option that a single job allows the worker eventually to switch employers or take on a second job. Second, the employer's component $S_j^+(y_1, z)$ shown in Equation (19) captures, for the incumbent firm, the effect of becoming the primary employer if the worker takes on a second job, the net surplus of which is $J_1(y_1, y_2, z) - (1 - \phi) S(y_1, z)$. Third, if the worker neither leaves nor takes on a second job, then in Equation (17) the worker-firm pair receives the surplus $S(y_1, z)$ in the subsequent period if the job is kept alive.

Next, consider the match surplus between a secondary employer and a multiple jobholder, $S(y_1, y_2, z)$. Its asset value is given by

$$S(y_1, y_2, z) = y_2 f(h(y_1, y_2, z)) + zg(1 - h(y_1, z) - h(y_1, y_2, z)) - \omega_2$$
$$- (\phi S(y_1, z) + N(z) + \omega_1 - w(y_1, z)) + \beta \Bigg(S_e^+(y_1, y_2, z) + \int \left(\int \int p(y_1', z') \right.$$
$$\times d(y_1', y_2', z') S(y_1', y_2', z') dF(y_1'|y_1) dF(y_2'|y_2)$$
$$+ \left(\int (1 - p(y_1', z')) dF(y_1'|y_1) \right) \left(\int p(y_2', z') S(y_2', z') dF(y_2'|y_2) \right) \Bigg) dG(z'|z) \Bigg) \quad (20)$$

where
$$S_e^+(y_1, y_2, z) = \int \left(N(z') + \phi \int p(y_1', z') S(y_1', z') dF(y_1'|y_1) \right) dG(z'|z). \quad (21)$$

In the continuation value of $S(y_1, y_2, z)$, $S_e^+(y_1, y_2, z)$ defined in Equation (21) captures the option value of the worker returning to single jobholding at her primary employer, which happens

when the worker terminates the multiple jobholding spell. The remaining part in Equation (20) shows that the employment relationship may continue as a spell of multiple jobholding or evolve into single employment at the secondary employer (who would then become the sole employer of the worker).

Last, the asset value of being the primary employer of a multiple jobholder solves

$$J_1\left(y_1, y_2, z\right) = y_1 f\left(h\left(y_1, z\right)\right) - w\left(y_1, z\right) + \beta \int \int p\left(y_1', z'\right)\left(\left(1-\phi\right) S\left(y_1', z'\right) + \int\left(d\left(y_1', y_2', z'\right)\right.\right.$$

$$\times \left.\left.\left(J_1\left(y_1', y_2', z'\right) - \left(1-\phi\right) S\left(y_1', z'\right)\right) dF\left(y_2'|y_2\right)\right) dF\left(y_1'|y_1\right) dG\left(z'|z\right). \quad (22)$$

That is, in the following period the employer becomes the only employer of the worker if the worker gives up her second job. Otherwise, she continues as her primary employer and receives the net surplus value $J_1\left(y_1, y_2, z\right) - \left(1-\phi\right) S\left(y_1, z\right)$.

At this point, it is useful to compare $J_1\left(y_1, y_2, z\right)$ and $\left(1-\phi\right) S\left(y_1, z\right) = J\left(y_1, z\right)$ – its full expression is in Appendix A.3. First, the profit flows $y_1 f\left(h\left(y_1, z\right)\right) - w\left(y_1, z\right)$ are the same due to Assumption (A2). Second, in the continuation value of $J_1\left(y_1, y_2, z\right)$, $\left(1-\phi\right) S\left(y_1, z\right)$ is only subjected to $p\left(y_1, z\right) = 1$ – the job remains viable –, whereas in the continuation value of $J\left(y_1, z\right)$ it is also multiplied by $1 - \lambda_1 + \lambda_1\left(1 - \ell\left(y_1, y_2, z\right)\right) \leq 1$ – the worker did not receive an outside job offer, or she received one but rejected it. Third, $J_1\left(y_1, y_2, z\right) - \left(1-\phi\right) S\left(y_1, z\right)$ in Equation (22) is multiplied by $p\left(y_1, z\right) d\left(y_1, y_2, z\right)$ whereas in the continuation value of $J\left(y_1, z\right)$ it is multiplied by $\lambda_1\left(1 - \ell\left(y_1, y_2, z\right)\right) p\left(y_1, z\right) d\left(y_1, y_2, z\right) \leq 1$. These expressions are the direct product of Assumption (A1). Given that the profit flows are the same and that job turnover is higher for single vs. multiple jobholders, we would expect $J_1\left(y_1, y_2, z\right) \geq J\left(y_1, z\right)$ to hold. The reason we cannot guarantee this result *ex ante* is that y_2' is drawn from $F\left(y_2'|y_2\right)$ in the continuation value of $J_1\left(y_1, y_2, z\right)$, whereas it is drawn from $F_0\left(.\right)$ in the continuation value of $J\left(y_1, z\right)$. If $F_0\left(.\right)$ first-order stochastically dominates $F\left(.|y_2\right)$, then this could reverse the comparison between $J_1\left(y_1, y_2, z\right)$ and $J\left(y_1, z\right)$. This never happens in the equilibrium of the calibrated model, however. In this equilibrium, $F_0\left(.\right)$ dominates $F\left(.|y_2\right)$ only for values of match productivity y_2 that are lower than average match productivity, but at these values of y_2 we have $d\left(y_1, y_2, z\right) = 0$ since the second job does not generate enough surplus to cover ω_2.

To compute the joint surplus value of multiple jobholding (Equation (20)), we need to determine the wage of a single jobholder, $w\left(y_1, z\right)$. From the asset value of employing a single jobholder, it follows that

$$w\left(y_1, z\right) = y_1 f\left(h\left(y_1, z\right)\right) - \left(1-\phi\right) S\left(y_1, z\right) + \beta\left(S_j^+\left(y_1, z\right) + \left(1-\phi\right)\right.$$

$$\times \left.\int\left(\int p\left(y_1', z'\right)\left(1 - \lambda_1 \int \ell\left(y_1', y_2', z'\right) dF_0\left(y_2'\right)\right) S\left(y_1', z'\right) dF\left(y_1'|y_1\right)\right) dG\left(z'|z\right)\right) \quad (23)$$

for all y_1 and z. As anticipated, $w\left(y_1, z\right)$ includes the expected value of the worker becoming a multiple jobholder without changes to her bargaining position vis-à-vis the primary employer, through the term $S_j^+\left(y_1, z\right)$ (see Equation (19)). We can also recover $w\left(y_1, y_2, z\right)$, the wage of

a multiple jobholder, by using the asset value of secondary employers. $w(y_1, y_2, z)$ is given by

$$
w(y_1, y_2, z) = y_2 f(h(y_1, y_2, z)) - (1 - \phi) S(y_1, y_2, z) + \beta (1 - \phi) \int \left(\int \int p(y_1', z') \right.
$$

$$
\times d(y_1', y_2', z') S(y_1', y_2', z') dF(y_1'|y_1) dF(y_2'|y_2)
$$

$$
+ \left(\int (1 - p(y_1', z')) dF(y_1'|y_1) \right) \left(\int p(y_2', z') S(y_2', z') dF(y_2'|y_2) \right) \Big) dG(z'|z) \quad (24)
$$

for all y_1, y_2 and z.

3.5 Free entry condition

To write the free entry condition, we let $\varphi_0(z)$ and $\varphi_1(y_1, z)$ denote the population measure of nonemployed workers and single jobholders, respectively. Below we will denote by $\varphi_2(y_1, y_2, z)$ the population measure of multiple jobholders. The free entry condition yields

$$
\frac{\kappa}{q(\theta)} = \beta (1 - \phi) \left(\int \int p(y_1', z') S(y_1', z') dF_0(y_1') dG(z'|z) \frac{\varphi_0(z)}{\bar{\varphi}_0 + s_e \bar{\varphi}_1} dz \right.
$$

$$
+ \int \int \int S_j^+(y_1', y_2', z') dF_0(y_2') dF(y_1'|y_1) dG(z'|z) \frac{s_e \varphi_1(y_1, z)}{\bar{\varphi}_0 + s_e \bar{\varphi}_1} dy_1 dz \Bigg) \quad (25)
$$

where

$$
S_j^+(y_1, y_2, z) = \ell(y_1, y_2, z) p(y_2, z) S(y_2, z)
$$

$$
+ (1 - \ell(y_1, y_2, z)) p(y_1, z) d(y_1, y_2, z) S(y_1, y_2, z). \quad (26)
$$

$(1 - \phi) S_j^+(y_1, y_2, z)$ denotes the asset value of an employer with a vacant position who meets an employed worker. She takes as given the decision of the worker to leave the previous employer or to combine the two jobs together. In Equation (25), $\bar{\varphi}_0$ is the cumulated measure of nonemployed workers, i.e. $\bar{\varphi}_0 = \int \varphi_0(z) dz$. Likewise, $\bar{\varphi}_1$ is the cumulated measure of single jobholders. $\bar{\varphi}_0 + s_e \bar{\varphi}_1$ gives the number of job seekers, which is used to obtain the conditional distribution on the right-hand side of Equation (25) (and to compute tightness $\theta = {}^v/_{\bar{\varphi}_0 + s_e \bar{\varphi}_1}$, with v denoting the measure of vacancies).

3.6 Equilibrium

The steady-state equilibrium of this economy is defined as follows:

Definition. A steady-state equilibrium is a list of asset values $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$; a list of wage schedules $w(y_1, z)$, $w(y_1, y_2, z)$ and schedules of hours worked $h(y_1, z)$, $h(y_1, y_2, z)$; a list of policy functions for match formation and continuation, $p(y_1, z)$, multiple jobholding $d(y_1, y_2, z)$ and quit decisions $\ell(y_1, y_2, z)$; a population distribution $\varphi_0(z)$, $\varphi_1(y_1, z)$, $\varphi_2(y_1, y_2, z)$; and a value of tightness θ such that:

1. Given wages $w(y_1, z)$, $w(y_1, y_2, z)$ and schedules of hours $h(y_1, z)$, $h(y_1, y_2, z)$, the policy functions $p(y_1, z)$, $d(y_1, y_2, z)$, $\ell(y_1, y_2, z)$, and tightness θ, the asset values $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$ solve the Bellman equations that sum up to (17), (20) and (22) through the surplus sharing Equations (8) and (9).

2. Given the asset values $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$, and tightness θ, the wage schedules $w(y_1, z)$, $w(y_1, y_2, z)$ yield the surplus sharing equations (8) and (9), and the schedules of hours worked $h(y_1, z)$, $h(y_1, y_2, z)$ solve Equations (10) and (11), respectively.

3. Given the asset values $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_2(y_1, y_2, z)$ combined into joint surpluses via Equations (4) and (5), the policy functions $p(y_1, z)$, $d(y_1, y_2, z)$, $\ell(y_1, y_2, z)$ are given by Equations (13), (14) and (15), respectively.

4. Given the policy functions $p(y_1, z)$, $d(y_1, y_2, z)$, $\ell(y_1, y_2, z)$, and tightness θ, the population distribution $\varphi_0(z)$, $\varphi_1(y_1, z)$, $\varphi_2(y_1, y_2, z)$ is time invariant with respect to the set of stock-flow equations of the economy.

5. Given the asset values $J(y_1, z)$ and $J_2(y_1, y_2, z)$ combined into joint match surpluses through Equations (8) and (9), and population distribution $\varphi_0(z)$, $\varphi_1(y_1, z)$, labor-market tightness θ solves Equation (25).

The stock-flow equations across the different states of nature (condition 4 in the above definition) can be deduced from the model's description. Given a population distribution $\varphi_0(z)$, $\varphi_1(y_1, z)$, $\varphi_2(y_1, y_2, z)$ and a value of market tightness θ, Proposition 2 enables us to compute the equilibrium by solving Equations (17), (20), (22) (for instance using value-function iterations) while recovering the wage schedule $w(y_1, z)$ via Equation (23).

4   Quantification and model validation

Next, we turn the theoretical framework presented in the previous section into a fully quantitative tool. This is a key step of our analysis, given that the motivation and the questions that it addresses are quantitative in nature.

4.1   Specification

We begin with the specification of stochastic processes. As is standard, we assume that match productivity y evolves according to a first-order autoregressive process. We denote by μ_y the unconditional mean of the process, $\rho_y \in (0, 1)$ the persistence, and σ_ε^2 the variance of the innovation term. Given a pre-specified number of grid points to discretize the support of match productivity, we approximate the transition function $F(y'|y)$ using Rouwenhorst [1995]'s method. For simplicity we use $F_0(\cdot) = F(\cdot|\mu_y)$ for the distribution from which y is drawn upon meeting. Next, we need a stochastic process for home productivity, z. We use a first-order

Markov process defined in the following way: with probability ρ_z the value of z is unchanged, while with probability $1 - \rho_z$ a new z' is drawn from a Normal distribution $\mathcal{N}\left(\mu_z, \sigma_z^2\right)$.[17] We discretize and truncate this distribution to the interval $[\mu_z - 2\sigma_z, \mu_z + 2\sigma_z]$.

Next, we need a production function for the home good, $g\left(.\right)$, and a function mapping a worker's hours worked to her market's labor services, $f\left(.\right)$. For the former, we use a standard specification that we can relate to studies from the labor supply literature:

$$g\left(1 - h\right) = \frac{\left(1 - h\right)^{1 - \frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}}. \tag{27}$$

For $f\left(.\right)$, we propose a less standard, but intuitive and parsimonious specification. We use the following piecewise linear function:

$$f\left(h\right) = \begin{cases} \left(1 - \psi\right)h & \text{if } h < \bar{h} \\ \left(1 - \psi\right)h + \psi & \text{if } h \geq \bar{h} \end{cases}, \tag{28}$$

where $\psi \geq 0$ and $\bar{h} > 0$ are exogenous parameters. This simple construct generates a meaningful distinction between full-time and part-time employment (later defined as hours worked below the $\bar{h}$ threshold) that will prove very useful for making connections to the data.[18] Indeed, in (28), the marginal increment in labor services coming from additional hours worked is constant except for some neighborhood to the left of $\bar{h}$. Since a small increase in hours worked in that neighborhood (the cost of which is a small reduction in home production) generates a discrete increase by ψ units in labor services, this should 'bunch' working hours towards $\bar{h}$. The Proposition below confirms this intuition.

Proposition 3. *The hours schedule in a primary job is given by:*

$$h\left(y_1, z\right) = \begin{cases} \bar{h} & \text{if } y_{\bar{h}}\left(z\right) \leq y_1 < \widetilde{y}\left(z\right) \\ 1 - \left(\frac{z}{\left(1 - \psi\right)y_1}\right)^{\gamma} & \text{otherwise,} \end{cases} \tag{29}$$

for all (strictly positive) y_1 and z. $y_{\bar{h}}\left(z\right)$ and $\widetilde{y}\left(z\right)$ are functions presented in the Appendix.

The hours schedule for the second job of multiple jobholders is:

$$h\left(y_1, y_2, z\right) = \begin{cases} \bar{h} & \text{if } y_{\bar{h}}\left(y_1, z\right) \leq y_2 < \widetilde{y}\left(y_1, z\right) \\ 1 - h\left(y_1, z\right) - \left(\frac{z}{\left(1 - \psi\right)y_2}\right)^{\gamma} & \text{otherwise,} \end{cases} \tag{30}$$

for all (strictly positive) y_1, y_2, and z. $y_{\bar{h}}\left(y_1, z\right)$ and $\widetilde{y}\left(y_1, z\right)$ are functions presented in the

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[17]In Lalé [2018], we use a similar shock process for workers' preferences for leisure to generate worker flows in and out of the labor force. From a computational standpoint, the advantage of this stochastic process is that conditional on changing z, the new z' is drawn independently from the previous z.

[18]The non-convex mapping from hours worked to labor services in (28) shares similarities with that proposed by Prescott et al. [2009], Rogerson and Wallenius [2009] and Chang et al. [2019] to distinguish between the extensive and intensive margins of labor adjustments. It also parallels with the mapping from hours to earnings studied in Bick et al. [2022].

Appendix.

Proof. See Appendix A.4. □

Consider the hours schedule of a single jobholder, or similarly that of the primary job of a multiple jobholder, described in (29). Hours worked increase in a concave fashion with match productivity y_1. At $y_1 = y_{\bar{h}}(z)$, agents are indifferent between setting hours worked at $1 - \left(\frac{z}{(1-\psi)y_{\bar{h}}(z)}\right)^\gamma < \bar{h}$ vs. setting hours at $\bar{h}$, and the hours schedule jumps up to $\bar{h}$. For any y_1 between $y_{\bar{h}}(z)$ and $\widetilde{y}(z)$, the hours schedule is flat as it is optimal to keep hours at $\bar{h}$. For any $y_1 \geq \widetilde{y}(z)$, hours increase again with y_1 through $h(y_1, z) = 1 - \left(\frac{z}{(1-\psi)y_1}\right)^\gamma$. Notice that hours decrease with home productivity, z, and that hours worked in the primary job $h(y_1, z)$ reduce hours worked in the second job, $h(y_1, y_2, z)$.

To complete the model's specification, we need a matching function to map market tightness into the job-filling probability. We use a standard Cobb-Douglas matching function, such that the number of contacts per unit of time is $\chi v^{1-\alpha}(\bar{\varphi}_0 + s_e\bar{\varphi}_1)^\alpha$, where v denotes vacancies and $\bar{\varphi}_0 + s_e\bar{\varphi}_1$ is the number of job seekers. The job-filling probability is then $q(\theta) = \chi\theta^{-\alpha}$.

4.2 Calibration

For reasons discussed further below, we add to the model some exogenous job separations to discipline certain parameters. We assume that all jobs are destroyed with a per-period probability δ; conditional on not being hit by the δ shock, the events unfold as described in Section 3. Given these specification choices, the model has eighteen parameters:

$$\beta, \ \bar{h}, \ \mu_y, \ \rho_y, \ \alpha, \ \phi, \ \chi, \ \gamma, \ \mu_z, \ \rho_z, \ \sigma_z, \ \psi, \ \kappa, \ \sigma_\varepsilon, \ \delta, \ s_e, \ \omega_1, \ \omega_2.$$

We use external information to select parameter values for the first seven of these parameters. We calibrate the other parameters to match several data moments, most of which are based on CPS data analyzed in Appendix B. Throughout the analysis, the model period is set to be one month.

Externally calibrated parameters. We use a discount factor β of 0.9951 to accord with an annualized real interest rate of 6 percent. To choose $\bar{h}$, we note there are about 100 hours of substitutable time per week and the standard full-time work schedule amounts to 40 hours of work per month. So, we set $\bar{h} = 0.40$ given that the time endowment of workers has been normalized to 1 (Equation (27)). We can have one more normalization, namely the unconditional mean of match productivity, μ_y, which we set to 1. The next parameter is the persistence of match productivity, ρ_y. Following much of the literature, we interpret the empirical observation that wage shocks are highly persistent as evidence that match productivity is close to unit root. In practice we set $\rho_y = 0.975$. At an annual frequency, this implies a persistence of $0.975^{12} = 0.738$, which falls well within the range of estimates of wage processes.[19] We set the

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[19]See for instance in Table 1 in Chang and Kim [2006]: the authors report an annual persistence of wage shocks (estimated from PSID data) of 0.781 for men and 0.724 for women.

Table 1: Externally calibrated parameters

Parameter	Description	Value
β	Monthly discount factor	0.995
$\bar{h}$	Threshold for full-time work	0.4
μ_y	Match prod., uncond. mean	1.0
ρ_y	Match prod., persistence	0.975
α	Tightness elasticity of job filling prob.	0.5
ϕ	Workers' Nash bargaining share	0.5
χ	Matching efficiency	0.50

Notes: The table describes the model parameters that are based on external calibration. The model period is set to be one month.

elasticity of the vacancy-filling probability with respect to labor market tightness α and the bargaining power of workers ϕ equal to 0.5. Finally, the matching efficiency parameter χ is set to 0.50. Below we calibrate the vacancy-posting cost and use the free entry condition to pin down labor market tightness. Table 1 provides the list of externally calibrated parameters.

Internally calibrated parameters. The remaining parameters, namely γ, μ_z, ρ_z, σ_z, ψ, κ, σ_ε, δ, s_e, ω_1, ω_2, are calibrated to match eleven data moments. Although these are jointly determined, each parameter is more directly identified by a specific data moment, as we discuss momentarily. Given the strong relation between multiple jobholding and full-time/part-time employment on the one hand, and the relation between part-time employment and gender on the other, we calibrate the parameters separately for men (M) and women (W). We focus on individuals aged 25 to 54 years old with some College or higher education to reduce the amount of heterogeneity in the underlying data. For each gender group, we target:

1. The **Frisch elasticity of labor supply**. Given (27), the Frisch elasticity is

$$\mathcal{F} = \gamma \frac{1-h}{h}. \tag{31}$$

   We consider different values for $\mathcal{F}$, namely we use $\mathcal{F} = 0.45$ as our benchmark but we also study $\mathcal{F} = 0.30$ and $\mathcal{F} = 0.60$ to cover the range of plausible values (see Table 3C in Meghir and Phillips [2010] or Table 1 in Keane [2011]). Besides the uncertainty as to the precise value of the Frisch elasticity, we think that exploring the effects of different values of $\mathcal{F}$ is interesting in its own rights.

2–5. The **part-time employment share**, the **transition rate from full- to part-time** work hours, **average hours per worker**, and the share of workers **bunching at full-time hours**. We target four data moments on hours worked, three of which describe the cross-sectional distribution of working hours and one describing worker transitions within this distribution. Foremost, our motivation is that we need moments besides the mean and variance to characterize the empirical distribution of working hours, given that it has multiple modes, clustering, etc. (Borowczyk-Martins and Lalé [2019]). We know from Proposition 3 that the hours schedules are negatively related to z. This suggests that

the part-time employment share (defined in the model as the share of employed agents with strictly less than $\bar{h}$ total hours worked) and average hours per worker can be used to identify the mean value and dispersion of home productivity, μ_z and σ_z. Intuitively, the persistence of home productivity ρ_z matters for the rate at which workers move between full-time and part-time hours. As for the share of workers bunching at $\bar{h}$ hours, it is directly related to ψ for reasons also discussed in Proposition 3. Note that gender differences in μ_z, ρ_z, σ_z should emerge from this given that the part-time employment share is more than 3 times higher for female than for male workers, women transition at a twice higher rate between full-time and part-time working hours, and the gender gap in hours per worker is by 5 hours on average.

6. The **vacancy posting cost**. We follow Elsby and Michaels [2013] and calibrate κ so that the expected vacancy posting cost amounts to 14 percent of average quarterly earnings.[20]

7. The **employment separation rate**. We target the rate at which workers separate from employment, i.e. make a transition from employment to nonemployment.[21] In CPS data analyzed in Appendix B, we find that the monthly employment separation rate is 1.79 percent for men vs. 2.63 percent for women, i.e. a 45 percent higher rate. This data moment helps to identify the volatility of shocks to match productivity measured by σ_ε.

8. The **share of exogenous employment separation**. In this section, we have introduced some exogenous separation through δ, in addition to the endogenous separations that are triggered by shocks to match and/or home productivity. Without this ingredient, we need larger shocks to productivity to match the target for the employment separation rate, facing the risk that this pushes match productivity (or home productivity, since the underlying parameters are calibrated jointly) into negative territory. To calibrate δ, we assume that exogenous separations account for one fourth of all employment separations. This target pushes σ_ε down towards values similar to those in Bils et al. [2012].

9. The **job-to-job transition rate**. We calculate the share of employed workers who report a change in employer between two consecutive months of CPS interviews. We use it to calibrate on-the-job search efficiency, s_e.

10. The **employment rate**. We calculate the average employment rate over the time period covered by our CPS data (1994 until 2020) among workers who have been employed for at least one week in the previous calendar (this information is obtained from the March supplements of the CPS). We add this sample restriction to exclude prime-age individuals who are long term inactive and who are therefore not well captured by our model. We

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[20]Elsby and Michaels [2013] use empirical evidence based on Silva and Toledo [2009] to calculate this number.

[21]We purposely refrain from calling it the "job separation rate", although this terminology is more common place in the literature. The reason is that there are several types of job separation in addition to transitions from employment to nonemployment: job separations triggered by a job-to-job transition, and transitions from multiple into single jobholding as they entail a separation from either the primary or second job. The term "employment separation rate" is a better descriptor of the moment that we target.

Table 2: Internally calibrated parameters

Parameter	Description		Value	Targeted moment		Data	Model
γ	Curvature of $g(1-h)$	**M**	0.350	Frisch elasticity of labor supply*	**M**	0.45	0.45
		W	0.281		**W**	0.45	0.45
μ_z	Home prod., uncond. mean	**M**	0.260	Part-time empl. share	**M**	4.92	4.95
		W	0.238		**W**	17.6	17.5
ρ_z	Home prod., persistence	**M**	0.818	Full- to part-time trans. rate	**M**	1.29	1.30
		W	0.938		**W**	3.13	3.16
σ_z	Home prod., standard dev.	**M**	0.121	Average hours per worker	**M**	43.8	43.9
		W	0.087		**W**	38.4	38.2
ψ	Prod. gap at $\bar{h}$ hours	**M**	0.044	Share bunching at full-time hours	**M**	44.2	44.1
		W	0.026		**W**	45.1	44.9
κ	Vacancy posting cost	**M**	0.077	Exp. vacancy cost / qrtly earnings*	**M**	14.0	13.9
		W	0.067		**W**	14.0	14.0
σ_ε	Match prod., standard dev.	**M**	0.061	Empl. separation rate	**M**	1.79	1.80
		W	0.109		**W**	2.63	2.58
δ	Separation shock	**M**	0.005	Share exogenous empl. separation*	**M**	25.0	25.0
		W	0.007		**W**	25.0	25.0
s_e	On-the-job search efficiency	**M**	0.314	Job-to-job transition rate	**M**	1.71	1.71
		W	0.311		**W**	1.75	1.77
ω_1	Cost of working job 1	**M**	0.134	Employment rate	**M**	95.0	94.9
		W	0.126		**W**	93.5	93.6
ω_2	Cost of working job 2	**M**	0.043	Multiple jobholding share	**M**	6.57	6.58
		W	0.077		**W**	6.62	6.76

Notes: The table describes the model parameters (left panel) that provide the best fit to the data (right panel). M and W denote model and data moments for respectively men and women. The model period is set to be one month. The data moments except those marked with an asterisk are based data from the Current Population Survey for individuals aged 25 to 54 with some College or higher education; those marked with an asterisk are taken from the literature. All moments except the Frisch elasticity of labor supply and average hours per worker are expressed in percent.

obtain employment rates of 95.0 and 93.5 percent for men and women, respectively. We use these as calibration targets for the flow cost of working on the first job, ω_1.

11. The **employment share of multiple jobholders**. We target the multiple jobholding share to calibrate the flow cost of working a second job, ω_2. It is important to point out that this is the only targeted data moment that concerns multiple jobholding.

Table 2 presents the outcome of the calibration. The model attributes the difference in working hours between men and women to a combination of differences in the volatility and persistence of productivity in the home sector (σ_z and ρ_z), and to a lesser extent to its mean productivity (μ_z). These parameters depend much on the Frisch elasticity of labor supply. This is illustrated by Table C1 in the appendix reporting the results of the calibration for $\mathcal{F} = 0.30$ and $\mathcal{F} = 0.60$, and appendix Figure C1 showing the distribution of z for these two calibrations as well as the baseline case ($\mathcal{F} = 0.45$). The other main gender difference in Table 2 lies in the cost of working a second job, ω_2. To account for the fact that women have a similar multiple jobholding share compared to men while being much more likely to work part time, the model sets ω_2 to a higher value.

At the same time, the calibration selects similar values of on-the-job search efficiency for men and women, namely $s_e = 0.31$. This value is somewhat higher than in on-the-job search models of the kind studied by Jolivet et al. [2006], but not too far off from Mukoyama [2014]

who uses $s_e = 0.25$. Notice that in this model, not all on-the-job contacts result in a job-to-job transition: a bad draw of match productivity y_2 would make the worker ignore the outside employer, and for some draws of y_2 the worker becomes a multiple jobholder. In fact, the value of s_e computed in Table 2 is potentially more reliable than many available in the literature because it takes accounts of both job-to-job transitions and multiple jobholding.

4.3 Model fit

The last column of Table 2 shows that the model matches all targeted data moments very well. It does so for both men and women. Table C1 in the appendix shows that the model does equally well when we use a different curvature of the home production function.

Untargeted monthly data moments. Table 3 puts the model to a more stringent test. It compares a set of worker flows generated by the model with their empirical counterparts, all of which are not targeted by the model's calibration.[22]

Table 3: Comparison of monthly data and model moments

Untargeted moment		Data	Model
(a) Multiple jobholding inflow trans. prob.			
Full-time single to multiple jobholding	M	2.11	1.82
	W	1.97	1.79
Part-time single to multiple jobholding	M	5.79	4.34
	W	4.01	3.90
Nonemployment to multiple jobholding	M	0.60	0.00
	W	0.25	0.00
(b) Multiple jobholding outflow trans. prob.			
Full-time multiple to single jobholding	M	26.0	22.6
	W	26.4	22.8
Full-time multiple to nonemployment	M	0.52	0.15
	W	0.58	0.15
Part-time multiple to single jobholding	M	30.1	31.9
	W	31.1	32.6
Part-time multiple to nonemployment	M	2.81	0.80
	W	1.55	0.89
(c) Job-to-job trans. prob.			
Job-to-job trans. among single jobholders	M	1.65	1.51
	W	1.64	1.49
Job-to-job trans. among multiple jobholders	M	2.51	4.41
	W	3.26	5.34

Notes: The table reports model-generated moments and their counterparts based on data from the Current Population Survey for individuals aged 25 to 54 with some College or higher education. M and W denote model and data moments for respectively men and women. All table entries are expressed in percent.

Panels (a) and (b) of Table 3 show that, on the whole, the model does a good job at explaining worker flows in and out of multiple jobholding. First, it consistently predicts that multiple jobholding is more prevalent among individuals who are working part-time as opposed

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[22]See Appendix B for additional information on the calculation of data moments reported in Table 3.

to full-time on their primary job. That is, the inflow transition rate is about twice as high for part-time workers as for full-time workers. The model slightly understates the transition rate for moving from multiple to single jobholding with a full-time primary job. But it captures the fact that this transition probability is lower than the probability of moving to single jobholding among multiple jobholders with a part-time primary job. Second, the assumption that workers in the model cannot move directly from nonemployment to multiple jobholding is in line with the data. In the reverse direction, the model generates some transitions directly from multiple jobholding to nonemployment, and the order of magnitude is similar to that of the data.

Panel (c) of Table 3 compares job-to-job transition rates respectively for single and multiple jobholders. The model matches the average job-to-job transition rate (Table 2) by slightly underestimating that of single jobholders while overstating that of multiple jobholders. To understand these outcomes better, we analyze the sources of transitions from multiple to single jobholding. In the model, one in four of these transitions occur because the first job match is dissolved, meaning that the worker moves to the secondary employer and the transition counts as a job-to-job transition.[23] In the CPS data, we find that only about 5 percent of workers who switch from multiple to single jobholding between two consecutive months report a change of employer in their main job. Note that this observation strongly supports Assumption (A1), that a worker is tied to her primary employer.

Untargeted yearly data moments. Next, we analyze a set of data moments at the yearly frequency. Our motivation is twofold. First, by looking beyond averages of monthly transition rates, we get a better sense of the degree of individual heterogeneity that the model generates. Second, as illustrated by the study of Paxson and Sicherman [1996], the PSID provides a set of key annual data moments that complement labor market information from the CPS and that are useful to analyze multiple jobholding.[24]

To begin with, we use our model as a tool to simulate the trajectory of a large number of multiple jobholders over a long time period, so as to study the duration of completed spells of multiple jobholding. This distribution is rightly skewed, with a median duration of completed spells of 2 months and a mean duration of 4.1 months for women and 4.4 months for men. The latter number is different from the average expected duration of multiple jobholding that can be computed out of the hazard rate of transitioning out of multiple jobholding, due to the heterogeneity in transition rates.

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[23]There are two outcomes that account for the remaining 75 percent of transitions from multiple to single jobholding. In some instances (about 15 percent of the remaining transitions), the worker gives up a second job that, at that point, would not be viable even if it were matched to a single jobholder. That is, $J_2(y_1, y_2, z)$ (which is proportional to $E(y_1, y_2, z) - E(y_1, z)$) and $J(y_2, z)$ are both negative. This is likely to occur when y_2 suffers a large negative shock. But in most instances, the worker gives up a second job that would generate a positive surplus if it were matched to a single jobholder; the 'counterfactual' value $J(y_2, z)$ is positive.

[24]The March supplements of the CPS are not as useful for that matter. The March CPS records the number of weeks worked in the previous calendar year and the number of *usual* hours per week. There is a lot of heaping at 20 and 40 hours in this variable, which makes it unsuited for the calculation of total annual hours worked. Moreover, the March CPS does *not* record information on multiple jobholding. The survey counts the number of employers (up to three) in the previous calendar year, but it explicitly states that working for more than one employer simultaneously should be counted as only one employer.

Table 4: Comparison of yearly data and model moments

Untargeted moment		Data	Model
(a) Hours worked and multiple jobholding			
Total hours worked	**M**	2,260	2,203
	W	1,759	1,879
Multiple jobholding share	**M**	18.4	18.7
	W	20.9	20.7
(b) Changes in hours worked and multiple jobs			
Single jobholding $t-1$ and t	**M**	4.8 (452)	5.4 (541)
	W	29.1 (514)	-6.5 (455)
Single jobholding $t-1$; multiple jobholding in t	**M**	244 (673)	179 (912)
	W	262 (624)	125 (969)
Multiple jobholding $t-1$; single jobholding in t	**M**	-190 (649)	-33.1 (821)
	W	-223 (710)	-48.5 (891)
Multiple jobholding $t-1$ and t	**M**	3.6 (547)	9.2 (472)
	W	36.6 (677)	17.6 (393)

Notes: The table reports model-generated moments and their counterparts based on data from the Panel Study of Income Dynamics for individuals aged 25 to 54 with some College or higher education. M and W denote model and data moments for respectively men and women. Multiple jobholding shares are expressed in percent. All other data moments are expressed in hours. The numbers in parentheses are standard deviations.

We next simulate a panel dataset similar to the PSID and calculate yearly labor market outcomes based on those generated in each of the twelve months of simulated data. Panel (a) of Table 4 reports two key statistics: total annual hours worked, and the multiple jobholding share calculated over a 1-year horizon.[25] The model performs well at predicting total annual hours worked. This is perhaps not surprising, given the very good fit with regard to the distribution of weekly hours and to transitions in and out of employment. What is more remarkable is that it also matches the data very well with regard to the annual multiple jobholding share, around 18 percent for men and 21 percent for women. The empirical estimates are in line with Table 1 in Paxson and Sicherman [1996], with a few differences explained by sample selection (ours focuses on workers aged 25 to 54 years old with some College or higher education).[26]

To provide further evidence of the fit with regard to working hours, Figure 1 presents the model-generated distribution of total annual hours (Panel (a)) and its empirical counterpart from the PSID (Panel (b)). In the data for men, there is a fair amount of clustering at around

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[25]To calculate annual hours, recall that we interpret the model's time endowment as 100 hours per week that can be substituted between the market and production in the home sector.

[26]Table 1 in Paxson and Sicherman [1996] compares PSID and CPS data and shows rates of multiple jobholding that are about 4 times larger in the PSID, as a result of calculating the multiple jobholding share over a 1-year horizon. The discussion surrounding Table 1 of Paxson and Sicherman [1996] clarifies that the PSID is unlikely to confuse job-to-job transitions that happen over the course of the year with multiple jobholding.
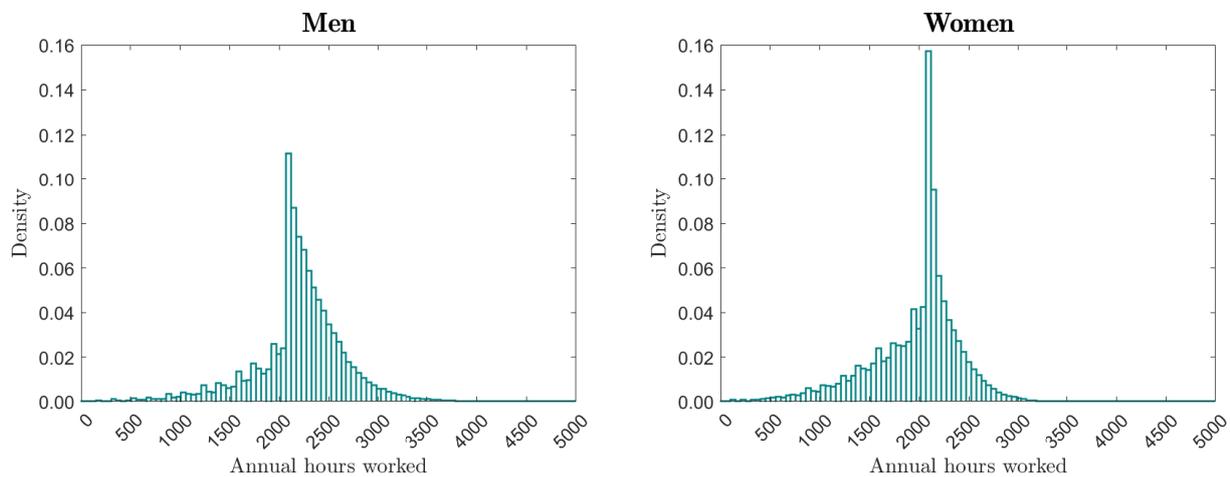
(a) Model
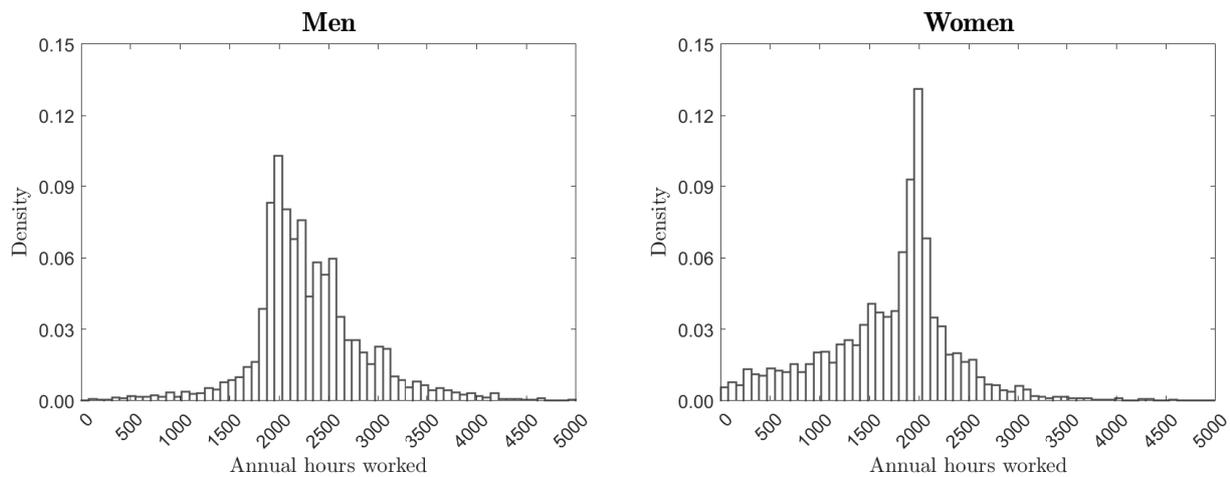
(b) Data

Figure 1: Total annual hours worked: Model vs. data

Notes: The panels in this figure plot, for men and women, the distribution of total annual hours worked. Panel (a) is based on simulated data from the baseline model (with parameter values matching a Frisch elasticity of labor supply of 0.45). Panel (b) is based on data from the Panel Study of Income Dynamics for individuals aged 25 to 54 with some College or higher education who report being employed in the previous calendar year.

2,000 hours, and the distribution is skewed to the right of this number. For women, there is also a mass point at 2,000 hours, but there is more mass to the left of the support. The model replicates all these patterns very well.[27]

Panel (b) of Table 4 turns to an important set of facts analyzed in Paxson and Sicherman [1996] concerning the relation between hours worked and multiple jobholding. Similar to Tables 8 and 9 and Figure 1 of their study, we calculate the mean and standard deviation of individual year-to-year changes in annual hours worked, conditional on different transitions (or lack of transition) over two consecutive years. Workers who remain single jobholders experience little change in hours on average. The model replicates this feature, as well as the wide dispersion around the mean observed in the data. It does equally well for workers who are multiple jobholders in two consecutive years. Multiple jobholding is an important channel for adjusting hours upwards. In the data, the average increase is around 250 annual hours. The model generates one half of this number for women, and almost 75 percent of it for men. In the reverse direction, workers give up a second job to reduce their working hours. The model is consistent with this pattern, but it generates a smaller portion of these changes in hours.

5 Determinants of multiple jobholding

In this section, we use the model as a tool to study the factors that affect the multiple jobholding share. We focus on the role of on-the-job search efficiency s_e and the flow cost of working a second job ω_2, as these parameters matter for accessing second jobs and for the duration of spells of multiple jobholding.

On-the-job search efficiency. Figure 2 reports the effects of changing on-the-job search efficiency on the (steady-state equilibrium) share of workers who have a second job. On the horizontal axis, we have normalized the value of s_e in the fully calibrated model to 1 to make the interpretation of the plots straightforward.[28] Figure 2 shows an interesting and perhaps counterintuitive finding: the multiple jobholding share is mostly flat, or is even increasing, when on-the-job search efficiency *decreases*. On the one hand, a lower s_e reduces the inflows of multiple jobholding because single jobholders become less likely to meet an outside employer. On the other, conditional on holding a second job, multiple jobholders are less willing to give it up because they anticipate that holding a second job in the future is less likely. As a result, the duration of spells of multiple jobholding lengthens. Given the ambiguous relation between multiple jobholding and s_e, it is quite remarkable that the model comes so close to the multiple jobholding inflows and outflows observed in the data.

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[27]The model misses somewhat the importance of long hours (over 3,500 annual hours of work) among men. There is a simple 'model fix' that consists in introducing an extra shock to home productivity such that z occasionally jumps to 0, making it optimal to devote all of agents' time endowment to market work. However, this extension does not bring in other insights to the model.

[28]The plots show the effects in models calibrated with different targets for the Frisch elasticity, and as a result the models have different parameter values for s_e. As can be seen, the results shown in Figure 2 are very robust to differences of the underlying calibration.
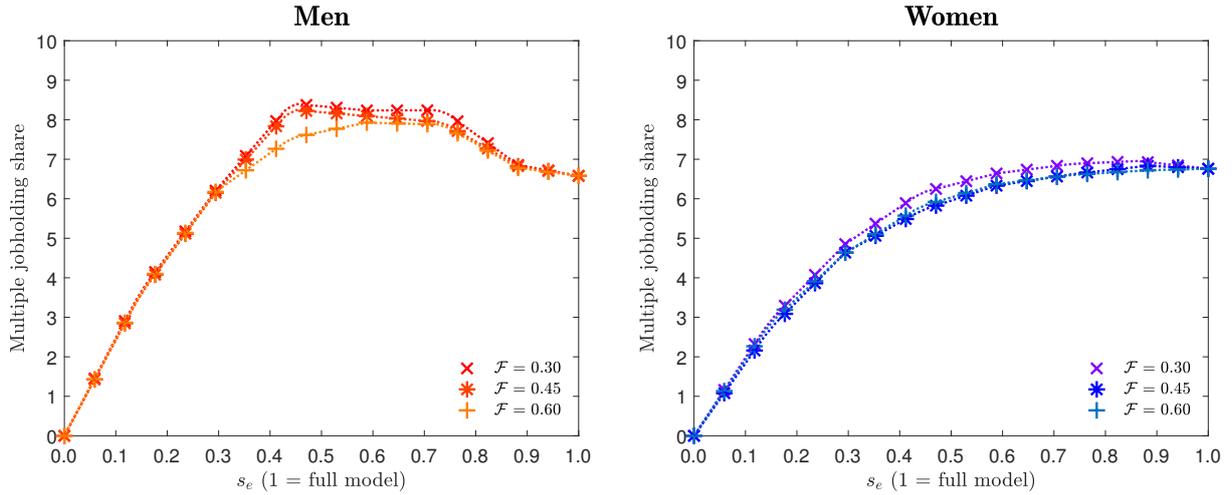
Figure 2: Effects of on-the-job search efficiency on multiple jobholding

Notes: The panels in this figure plot, for men and women, the multiple jobholding share computed for different values of on-the-job search efficiency, s_e. In the horizontal axes, 1 refers to the full calibrated model. $\mathcal{F}$ denotes the value of the Frisch elasticity of labor supply in the underlying calibration.

Flow cost of working a second job. The key parameter that allows the model to nail down the multiple jobholding share is ω_2, the flow cost of working a second job.[29] Table 5 compares its value to the average value of monthly earnings, denoted as $\bar{w}$. According to the model, the flow cost of working a second job represents 8 percent of the monthly earnings of men, and 15 percent of those of women. Given existing empirical estimates of the expenditures necessitated by work, this suggests that nonmonetary factors play a role in explaining the flow cost of working a second job.[30]

Table 5: Flow costs of working and the value of leisure

Moment	Description	Value		
		$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
$\frac{\omega_1}{\bar{w}}$	Cost of working job 1 / earnings			
	M	40.7	25.6	20.0
	W	40.9	23.2	16.3
		$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
$\frac{\omega_2}{\bar{w}}$	Cost of working job 2 / earnings			
	M	7.91	8.26	8.42
	W	15.1	14.7	16.7
		$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
$-\frac{\bar{\omega} + \bar{z}g}{\bar{w}}$	Value of leisure / earnings			
	M	79.0	72.9	78.9
	W	78.3	70.3	76.3

Notes: The table reports model-generated moments comparing the flow costs of working and value of leisure to earnings. M and W denote model moments for respectively men and women; $\mathcal{F}$ denotes the value of the Frisch elasticity of labor supply in the underlying calibration. All table entries are expressed in percent.

In Table 5, we also compare the different costs of working to average earnings to contrast them to ω_2 as well as compare our calibration with that of standard search models. Unlike ω_2,

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[29]We find a larger-than-unity elasticity of the multiple jobholding share with respect to ω_2. This finding resonates with the observation that different MSAs, which are likely characterized by different ω_2's, exhibit vastly different employment shares of multiple jobholders. As pointed out by [Hirsch et al., 2017, p.27], "*Differences in industry and occupation structure, commute times, job churn rates, and ancestry patterns explain a significant share of the [multiple jobholding share] variation across MSAs*".

[30]The empirical evidence presented in Aguiar and Hurst [2013] indicates that work-related expenses represent 5 percent of consumer spending.

the flow cost of working a first job, ω_1, varies with the Frisch elasticity of labor supply, and its value is in the range of 20 to 40 percent of average earnings. This order of magnitude is similar to that of UI benefits in standard job search models. In addition to these flow costs, the model features a value (or utility) of home production through $zg(1-h)$. Thus, the total flow value of leisure is the weighted sum of ω_1 and ω_2, denoted as $\bar{\omega}$, and the sum of $zg(1-h)$ weighted by the employment distribution of workers, which we denote as $\bar{z}g$. Table 5 shows that the flow value of leisure amounts to between 70 and 80 percent of average earnings, depending on the Frisch elasticity of labor supply. This order of magnitude is similar to the leisure flow value in calibrated version of the DMP model that produce realistic fluctuations of the unemployment rate (see Hall and Milgrom [2008], Robin [2011], Fujita and Ramey [2012]). The last remark about Table 5 is that $\omega_2 < \omega_1$, meaning that there are returns to scale in working.

6 Aggregate effects of multiple jobholding

We now turn to the main applications of the model. We use it to analyze the implications of multiple jobholding for the equilibrium outcomes of search models, and for the inference that these models provide with regard to labor market dynamics.

6.1 Comparison with a standard search model

A key property of our model is that it nests a simpler on-the-job search model in which employed workers cannot have more than one job – the standard setting of all search models. Furthermore, the model's calibration is such that none of the targeted moments depend on the number of jobs that workers have, except for the multiple jobholding share. That is, in Table 2, all data moments but the last one can be computed within the context of the search model that does away with multiple jobholding. We use these properties to analyze this 'nested' search model.

Table 6: Internally calibrated parameters (subset) in the model without multiple jobholding

Parameter	Description		Value		
			% change from full model		
			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
		M	0.039	0.043	0.045
			-23.1%	*-29.8%*	*-33.2%*
σ_ε	Match prod., standard dev.	**W**	0.071	0.101	0.146
			-6.3%	*-8.2%*	*-9.9%*
			$\mathcal{F} = 0.30$	$\mathcal{F} = 0.45$	$\mathcal{F} = 0.60$
		M	0.312	0.314	0.318
			+0.9%	*+1.1%*	*+1.2%*
s_e	On-the-job search efficiency	**W**	0.345	0.347	0.352
			+10.6%	*+11.6%*	*+13.1%*

Notes: The table describes a subset of the internally calibrated parameters of the model without multiple jobholding and compares (in italicized numbers) their values to the full model with multiple jobholding. M and W denote model moments for respectively men and women; $\mathcal{F}$ denotes the value of the Frisch elasticity of labor supply in the underlying calibration.

Table 6 summarizes the main differences between the calibrated parameters of the full model vs. the model without multiple jobholding; see Table C2 in the appendix for details. First,

shocks to match productivity are substantially less volatile in the model that abstracts from multiple jobholding. Depending on the Frisch elasticity of labor supply, the standard deviation of these shocks, σ_ε, is 23 to 33 percent lower in the calibration for male workers. For female workers, the order of magnitude is lower, with standard deviations that are 6 to 10 percent lower compared to the full model. Foreshadowing the discussion below, those differences reflect the fact that multiple jobholding lowers the employment separation rate. Second, the value of on-the-job search efficiency is higher in the model without multiple jobholding, but not by a large margin. For men, the order of magnitude of the difference is only 1 percent, while for women it is 11-13 percent higher.

We then start off from the calibrated model without multiple jobholding (Tables 6 and C2) and introduce the option of holding a second job for $\omega_2 = 0.1$, $\omega_2 = 0.2$, etc. This allows us to study the steady state equilibrium outcomes that are associated with different ω_2's, and hence with different levels of the multiple jobholding share.

Table 7: Effects of multiple jobholding on aggregate labor market outcomes

Moment		Value			
		% change from ref.			
Multiple jobholding share		0.00	2.50	5.00	7.50
Employment rate	M	95.0	95.2	95.4	95.6
		ref.	*+0.23%*	*+0.39%*	*+0.60%*
	W	93.6	93.8	94.1	94.3
		ref.	*+0.30%*	*+0.53%*	*+0.77%*
Tightness	M	0.50	0.49	0.49	0.48
		ref.	*-1.17%*	*-2.36%*	*-3.42%*
	W	0.67	0.66	0.65	0.65
		ref.	*-1.84%*	*-2.88%*	*-3.05%*
Empl. separation rate	M	1.78	1.69	1.62	1.55
		ref.	*-5.40%*	*-9.18%*	*-13.4%*
	W	2.63	2.48	2.37	2.27
		ref.	*-5.66%*	*-9.95%*	*-13.6%*
Job-to-job transition rate	M	1.73	1.74	1.75	1.76
		ref.	*+0.15%*	*+0.93%*	*-1.71%*
	W	1.76	1.81	1.83	1.85
		ref.	*+2.74%*	*+4.02%*	*+5.27%*
Average hours per worker	M	44.2	44.26	44.28	44.30
		ref.	*+0.04%*	*+0.10%*	*+0.12%*
	W	39.7	39.8	39.8	39.9
		ref.	*+0.10%*	*+0.22%*	*+0.35%*

Notes: The table reports moments from the model without multiple jobholding computed for values of the flow cost of working a second job, ω_2, that yield a multiple jobholding share of respectively 0%, 2.5%, 5%, and 7.5%. The italicized numbers compare values to those of the model without multiple jobholding (denoted as 'ref.'). M and W denote model moments for respectively men and women. The Frisch elasticity of labor supply in the underlying calibration is 0.45. All table entries except tightness and average hours per worker are expressed in percent.

Table 7 presents the results.[31] The main takeaway is that multiple jobholding has a non-negligible impact on the extensive margin (the number of workers employed), while it has little

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[31]In Table 7, we use the model with a Frisch elasticity of labor supply of 0.45. The results are qualitatively similar in the models calibrated to match $\mathcal{F} = 0.30$ or $\mathcal{F} = 0.60$.

effect on the intensive margin (hours per worker). Foremost, introducing multiple jobholding lowers the rate of employment separation: it becomes almost 15 percent lower when multiple jobholders account for a non-trivial share (7.5 percent) of employment. The reason for this is that multiple jobholding makes both the worker and the primary employer better off for a given match y_1 and home productivity z, resulting in fewer destructions of primary job matches. As a result, the employment rate increases. The order of magnitude is plausibly small, i.e. less than a 1 p.p. increase in aggregate employment. The employment gains would be larger if market tightness would remain constant, but as Table 7 shows, multiple jobholding has a negative impact on this variable. The last section discusses this relationship in more details. The last rows of Table 7 show that the effects of multiple jobholding on average hours per worker are small. It should be noted that there is a composition effect driving this result. Average hours worked in primary jobs decrease (since those jobs are kept alive for lower values of match productivity), and at the same time second jobs bring in additional hours per worker.

We now piece together the results from Tables 6 and 7. First, multiple jobholding lowers the employment separation rate; absent this mechanism, the standard search model underestimates the volatility of match productivity shocks σ_ε consistent with the rate of employment separations observed in the data. Second, multiple jobholding has two effects on the job-to-job transition rate. On the one hand, it lowers the rate because a fraction of on-the-job meetings lead to a spell of multiple jobholding as opposed to a job-to-job transition. On the other, it pushes up the job-to-job transition rate since, as we noted in Subsection 4.3, multiple jobholding introduces a new channel of job-to-job transitions, when the first job match is dissolved and the worker remains employed at the secondary employer. This second channel dominates, so that the full model with multiple jobholding matches the empirical job-to-job rate with a lower value of s_e. Vice versa, the standard search model requires a slightly higher s_e to match the data. The quantitative differences are important for σ_ε and less so for s_e.

6.2 Implications for job creation

To understand further the importance of multiple jobholding for search models, we analyze how it affects the inference that these models provide with regard to labor market dynamics. Specifically, we ask how multiple jobholding affects job creation through the cost of posting vacancies, "the lynchpin of a matching model" (Ljungqvist and Sargent [2007]). We address this question through an illustrative exercise in which we study the effects of an exogenous shift in the on-the-job search efficiency parameter, s_e. Two obvious motivations for this exercise is that the job-to-job transition rate contracts markedly during recessions (e.g., Shimer [2005]), and that job-to-job transitions have declined over time, at least relative to the mid-1990s (see Mukoyama [2014] and Fujita et al. [2020]). We confine ourselves to a steady-state analysis of the effects of s_e.

From a search model perspective, a higher s_e increases job creation. This effect is present in our models, but is substantially more muted in the full model which takes account of multiple jobholding. To understand why, we examine the job creation condition through the lens of a sim-

ple decomposition. Let us define $\boldsymbol{\Omega} = \{p\left(y_1, z\right), d\left(y_1, y_2, z\right), \ell\left(y_1, y_2, z\right), s_e, \varphi_0\left(z\right), \varphi_1\left(y_1, z\right)\}$ and $\boldsymbol{S} = \{S\left(y_1, z\right), S\left(y_1, y_2, z\right)\}$. We use these notations to define the expected surplus conditional on meeting, $\mathbb{E}\left(\boldsymbol{S}|\boldsymbol{\Omega}\right)$, given by:

$$
\begin{aligned}
\mathbb{E}\left(\boldsymbol{S}|\boldsymbol{\Omega}\right) = &\int \int p\left(y_1', z'\right) S\left(y_1', z'\right) dF_0\left(y_1'\right) dG\left(z'|z\right) \frac{\varphi_0\left(z\right)}{\bar{\varphi}_0 + s_e \bar{\varphi}_1} dz \\
&+ \int \int \int \left(\ell\left(y_1', y_2', z'\right) p\left(y_2', z'\right) S\left(y_2', z'\right) + \left(1 - \ell\left(y_1', y_2', z'\right)\right) p\left(y_1', z'\right)\right. \\
&\left. \times d\left(y_1', y_2', z'\right) S\left(y_1', y_2', z'\right)\right) dF_0\left(y_2'\right) dF\left(y_1'|y_1\right) dG\left(z'|z\right) \frac{s_e \varphi_1\left(y_1, z\right)}{\bar{\varphi}_0 + s_e \bar{\varphi}_1} dy_1 dz. \quad (32)
\end{aligned}
$$

Let v denote vacancies, and denote variables from the new equilibrium, viz. the equilibrium with a different on-the-job search efficiency, with an upper tilde ($\tilde{\ }$). We can decompose the change in vacancies from the baseline to the new equilibrium using the following relation:

$$
\begin{aligned}
\tilde{v} - v = &\underbrace{\left(\left(\tilde{\bar{\varphi}}_0 + \tilde{s}_e \tilde{\bar{\varphi}}_1\right) - \left(\bar{\varphi}_0 + s_e \bar{\varphi}_1\right)\right) \theta}_{\substack{\text{meeting probability} \\ \text{(extensive search margin)}}} \\
&+ \underbrace{\left(\frac{\chi}{\kappa} \beta\left(1 - \phi\right)\right)^{\frac{1}{\alpha}} \left(\mathbb{E}\left(\boldsymbol{S}|\tilde{\boldsymbol{\Omega}}\right)^{\frac{1}{\alpha}} - \mathbb{E}\left(\boldsymbol{S}|\boldsymbol{\Omega}\right)^{\frac{1}{\alpha}}\right) \left(\tilde{\bar{\varphi}}_0 + \tilde{s}_e \tilde{\bar{\varphi}}_1\right)}_{\substack{\text{matching | meeting} \\ \text{(intensive search margin)}}} \\
&+ \underbrace{\left(\frac{\chi}{\kappa} \beta\left(1 - \phi\right)\right)^{\frac{1}{\alpha}} \left(\mathbb{E}\left(\tilde{\boldsymbol{S}}|\tilde{\boldsymbol{\Omega}}\right)^{\frac{1}{\alpha}} - \mathbb{E}\left(\boldsymbol{S}|\tilde{\boldsymbol{\Omega}}\right)^{\frac{1}{\alpha}}\right) \left(\tilde{\bar{\varphi}}_0 + \tilde{s}_e \tilde{\bar{\varphi}}_1\right)}_{\substack{\text{surplus | matching} \\ \text{(joint surplus sharing)}}}. \quad (33)
\end{aligned}
$$

Consider the effects of having a higher s_e. The first term in (33) is the effect on the extensive margin of search. The number of job seekers increases, in search efficiency units, with s_e. Through the matching function, this creates a positive externality on the decision of firms to open more vacancies (since the probability of meeting a job seeker increases). The second term measures the effect of the intensive margin of search. On meeting a job seeker, there is a higher chance that this worker has a job, and hence a lower chance of matching conditional on meeting. This margin contributes negatively to vacancy creation. Third, the surplus of the worker is higher when on-the-job search efficiency increases. Since the joint surplus is shared, this third force has a positive impact on incentives for vacancy creation.

We use (33) to calculate the elasticity of job creation with respect to on-the-job search efficiency. Table 8 reports the results. In response to a 1 percent increase in s_e, the number of posted vacancies rises by 0.7–0.8 percent through the extensive search margin; higher joint

Table 8: Elasticity of job creation with respect to on-the-job search efficiency

Description		Multiple jobholding: with	without
Effect on meeting prob.	**M**	+0.78	+0.80
	W	+0.75	+0.78
Effect on matching | meeting	**M**	-1.57	-1.25
	W	-1.70	-1.46
Effect on surplus | matching	**M**	+1.13	+0.88
	W	+1.20	+1.02
Total effect	**M**	+0.34	+0.44
	W	+0.24	+0.33

Notes: The table reports the elasticity of job creation (the number of posted vacancies) with respect to on-the-job search efficiency through three channels: probability of meeting, probability of matching conditional on meeting, and expected surplus conditional on matching; as well as their combined effects. M and W denote model moments for respectively men and women. The Frisch elasticity of labor supply in the underlying calibration is 0.45. All table entries are expressed in percent.

surplus of employment adds a boost of 1.1–1.2 percent; and the intensive margin of search counteracts a substantial fraction of these effects by decreasing vacancies by 1.6–1.7 percent. Combined, these effects imply a job creation elasticity in the range of 0.24-0.34.

The precise value of the elasticity is not on its own the focus of our interest. The main point is rather that the model without multiple jobholding predicts job creation elasticities that are about a third larger, in the 0.33-0.44 range. Table 8 shows that the main driver of this difference is the intensive margin of search, whose effect is amplified by multiple jobholding. In the full model, when an employer meets an employed worker, there is the additional risk that the worker would turn her into a secondary employer. Since for most y_2 and z we have $J(y_2, z) > J_2(y_1, y_2, z) (\geq 0)$, firms are better off if they can avoid matching with these workers, which leads to a more negative effect of the intensive margin of search. In turn, it implies that the standard search model typically overstates the impact of on-the-job search on the incentives for vacancy creation.

7 Conclusion

We develop a general equilibrium search model of multiple jobholding. Central to the model's mechanism is the asymmetry between the employer on a worker's main job and the outside employer providing the second job. It allows us to obtain theoretical results that compare hours worked between the main job and the second job, as well as hours worked by single vs. multiple jobholders. Most importantly, this asymmetry puts discipline on the rates at which workers transition in and out of second jobs, which can then be used to test the model against data on multiple jobholding flows. We show that the model performs well on this test.

We then turn the model into a fully quantitative tool to analyze the implications of multiple jobholding for the equilibrium outcomes of search models, and for the inference that these models provide with regard to labor market dynamics. If workers were allowed to take on

second jobs within an otherwise standard search model, the main jobs would survive longer, and as a result the rate of worker separation from employment would be lower. The flip side is that the standard model underestimates the volatility of shocks to match productivity that trigger separations from employment. It also requires a higher on-the-job search efficiency than that in a full model with multiple jobholding, given that second jobs create an additional channel of job-to-job transitions. Last, since multiple jobholders have a stronger outside option to bargain with the outside employer compared with workers without jobs, multiple jobholding dampens the positive feedback from higher on-the-job search efficiency onto additional job creation. The capacity of on-the-job search to amplify labor market fluctuations is therefore lower than suggested by a standard search model.

Our model offers many possibilities for future work. Two of these are especially important. First, there is a perennial policy debate on the design of standard workweek hours and overtime provisions, and how these could be used to increase employment while permitting more time for leisure and for home production. Given the rich interplay between hours worked, multiple jobholding, and extensive margin adjustments, our model is a natural tool to investigate the impact of these policies quantitatively. Second, empirical research that looks at the effects of income taxation on hours worked finds very large differences in labor supply elasticities when measured using the primary or secondary job. The underlying reasons, which often remain unclear, hold different implications for tax policies, and in particular for whether second jobs should be exempted from taxes. Our model is suitable for investigating these effects in greater detail. We leave these and other applications for future research.

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Appendices

A Model appendix

Appendices A.1 and A.2 contain the proofs of Propositions 1 and 2, respectively. Appendix A.3 presents the Bellman equations associated to $N(z)$, $E(y_1, z)$, $E(y_1, y_2, z)$, $J(y_1, z)$, $J_1(y_1, y_2, z)$, $J_2(y_1, y_2, z)$ and shows how to combine them with the policy functions from Proposition 2 and the surplus-sharing Equations (8) and (9) to arrive at Equations (17), (20) and (22) of the main text. Appendix A.4 contains the proofs of Proposition A.4.

A.1 Proof of Proposition 1

We derive the results for wages and hours in primary jobs; the steps of the proofs are analogous for second jobs. Since the marginal utility of market consumption equals the marginal cost of paying wages, the first-order condition of the Nash bargaining problem is

$$\phi \frac{1}{E(y_1, z) - N(z)} = (1 - \phi) \frac{1}{J(y_1, z)}.$$

Rearrange this equation and use $S(y_1, z) = J(y_1, z) + E(y_1, z) - N(z)$ to obtain Equation (8). If in addition $f(.)$ and $g(.)$ are differentiable, then the first-order condition for hours is

$$\phi \frac{z g'(1 - h(y_1, z))}{E(y_1, z) - N(z)} = (1 - \phi) \frac{y f'(h(y_1, z))}{J(y_1, z)}.$$

Together with the above first-order condition for wages, this yields $z g'(1 - h(y_1, z)) = y f'(h(y_1, z))$, i.e. Equation (10) of the Proposition.

A.2 Proof of Proposition 2

The first two policy functions, $p(y_1, z)$ and $d(y_1, y_2, z)$, are trivially related to joint match surpluses. We have

$$p(y_1, z) = \mathbb{1}\{J(y_1, z) \geq 0\} = \mathbb{1}\{(1 - \phi) S(y_1, z) \geq 0\} = \mathbb{1}\{S(y_1, z) \geq 0\}$$

and

$$d(y_1, y_2, z) = \mathbb{1}\{E(y_1, y_2, z) \geq E(y_1, z)\} = \mathbb{1}\{\phi S(y_1, y_2, z) \geq 0\} = \mathbb{1}\{S(y_1, y_2, z) \geq 0\}.$$

Next, we look at

$$\ell(y_1, y_2, z) = \mathbb{1}\{\max\{E(y_2, z), N(z)\} \geq \max\{E(y_1, z), E(y_1, z) \\ + p(y_1, z)(E(y_1, y_2, z) - E(y_1, z)), N(z)\}\}.$$

Subtracting $N(z)$ on both side of the inequality yields

$$\ell(y_1, y_2, z) = \mathbb{1}\{\max\{E(y_2, z) - N(z), 0\} \geq \max\{E(y_1, z) - N(z), E(y_1, z) \\ + p(y_1, z)(E(y_1, y_2, z) - E(y_1, z)) - N(z), 0\}\}.$$

On the one hand, we have

$$\max\left\{E\left(y_2,z\right)-N\left(z\right),0\right\}=\max\left\{\phi S\left(y_2,z\right),0\right\}=p\left(y_2,z\right)\phi S\left(y_2,z\right).$$

On the other, $E\left(y_1,z\right)-N\left(z\right)=\phi S\left(y_1,z\right)$ and $E\left(y_1,y_2,z\right)-E\left(y_1,z\right)=\phi S\left(y_1,y_2,z\right)$, and thus we have

$$\max\left\{E\left(y_1,z\right)-N\left(z\right),E\left(y_1,z\right)+p\left(y_1,z\right)\left(E\left(y_1,y_2,z\right)-E\left(y_1,z\right)\right)-N\left(z\right),0\right\}$$
$$=\max\left\{\phi S\left(y_1,z\right),\phi\left(S\left(y_1,z\right)+p\left(y_1,z\right)S\left(y_1,y_2,z\right)\right),0\right\}$$
$$=\max\left\{\max\left\{\phi S\left(y_1,z\right),\phi\left(S\left(y_1,z\right)+p\left(y_1,z\right)S\left(y_1,y_2,z\right)\right)\right\},0\right\}$$
$$=\max\left\{\phi S\left(y_1,z\right)+\max\left\{0,p\left(y_1,z\right)\phi S\left(y_1,y_2,z\right)\right\},0\right\}$$
$$=\max\left\{\phi S\left(y_1,z\right)+p\left(y_1,z\right)d\left(y_1,y_2,z\right)\phi S\left(y_1,y_2,z\right),0\right\}.$$

If $S\left(y_1,z\right)\geq0$ then $S\left(y_1,z\right)+p\left(y_1,z\right)d\left(y_1,y_2,z\right)S\left(y_1,y_2,z\right)\geq0$, so that we also have

$$\max\left\{\phi S\left(y_1,z\right)+p\left(y_1,z\right)d\left(y_1,y_2,z\right)\phi S\left(y_1,y_2,z\right),0\right\}=p\left(y_1,z\right)\phi\left(S\left(y_1,z\right)\right.$$
$$\left.+d\left(y_1,y_2,z\right)S\left(y_1,y_2,z\right)\right)$$

and we arrive at

$$\ell\left(y_1,y_2,z\right)=\mathbb{1}\left\{p\left(y_2,z\right)S\left(y_2,z\right)\geq p\left(y_1,z\right)\left(S\left(y_1,z\right)+d\left(y_1,y_2,z\right)S\left(y_1,y_2,z\right)\right)\right\}.$$

A.3 Bellman equations

The asset value of a non-employed worker is

$$N\left(z\right)=\beta\int\left(\left(1-\lambda_0\right)N\left(z'\right)+\lambda_0\int\max\left\{E\left(y_1',z'\right),N\left(z'\right)\right\}dF_0\left(y_1'\right)\right)dG\left(z'|z\right)$$
$$=\beta\int\left(N\left(z'\right)+\lambda_0\int\max\left\{E\left(y_1',z'\right)-N\left(z'\right),0\right\}dF_0\left(y_1'\right)\right)dG\left(z'|z\right)$$

and since $E\left(y_1,z\right)-N\left(z\right)=\phi S\left(y_1,z\right)$, we have

$$N\left(z\right)=\beta\int\left(N\left(z'\right)+\lambda_0\int\max\left\{\phi S\left(y_1',z'\right),0\right\}dF_0\left(y_1'\right)\right)dG\left(z'|z\right)$$
$$=\beta\int\left(N\left(z'\right)+\lambda_0\phi\int p\left(y_1',z'\right)S\left(y_1',z'\right)dF_0\left(y_1'\right)\right)dG\left(z'|z\right)$$

using the policy function $p\left(y_1,z\right)=\mathbb{1}\left\{S\left(y_1,z\right)\geq0\right\}$.

The asset value of a single jobholder is

$$E\left(y_1,z\right)=w\left(y_1,z\right)-\omega_1+zg\left(1-h\left(y_1,z\right)\right)+\beta\int\left(\left(1-\lambda_1\right)\int\max\left\{E\left(y_1',z'\right),\right.\right.$$
$$N\left(z'\right)\right\}dF\left(y_1'|y_1\right)+\lambda_1\int\int\max\left\{E\left(y_1',z'\right)+p\left(y_1',z'\right)\left(E\left(y_1',y_2',z'\right)-E\left(y_1',z'\right)\right),\right.$$
$$\left.\left.E\left(y_1',z'\right),E\left(y_2',z'\right),N\left(z'\right)\right\}dF_0\left(y_2'\right)dF\left(y_1'|y_1\right)\right)dG\left(z'|z\right)$$

$$= w(y_1, z) - \omega_1 + zg(1 - h(y_1, z)) + \beta \int \Big(N(z') + (1 - \lambda_1) \int \max \{E(y_1', z')$$

$$- N(z'), 0\} \, dF(y_1'|y_1) + \lambda_1 \int \int \max \{E(y_1', z') + p(y_1', z')(E(y_1', y_2', z') - E(y_1', z'))$$

$$- N(z'), E(y_1', z') - N(z'), E(y_2', z') - N(z'), 0\} \, dF_0(y_2') \, dF(y_1'|y_1) \Big) \, dG(z'|z).$$

Since $E(y_1, z) - N(z) = \phi S(y_1, z)$ and $E(y_1, y_2, z) - E(y_1, z) = \phi S(y_1, y_2, z)$ via the surplus-sharing Equations (8) and (9), it follows that

$$E(y_1, z) = w(y_1, z) - \omega_1 + zg(1 - h(y_1, z)) + \beta \int \Big(N(z') + (1 - \lambda_1)$$

$$\times \int \max \{\phi S(y_1', z'), 0\} \, dF(y_1'|y_1) + \lambda_1 \int \int \max \{\phi(S(y_1, z) + p(y_1, z) S(y_1, y_2, z)),$$

$$\phi S(y_1', z'), \phi S(y_2', z'), 0\} \, dF_0(y_2') \, dF(y_1'|y_1) \Big) \, dG(z'|z).$$

Last, using the policy functions and results from Appendix A.2 we obtain

$$E(y_1, z) = w(y_1, z) - \omega_1 + zg(1 - h(y_1, z)) + \beta \int \Big(N(z') + (1 - \lambda_1) \int \phi p(y_1', z')$$

$$\times S(y_1', z') \, dF(y_1'|y_1) + \lambda_1 \int \int \phi(\ell(y_1', y_2', z') p(y_2', z') S(y_2', z') + (1 - \ell(y_1', y_2', z'))$$

$$\times p(y_1', z')(S(y_1', z') + d(y_1', y_2', z') S(y_1', y_2', z'))) \, dF_0(y_2') \, dF(y_1'|y_1) \Big) \, dG(z'|z)$$

given that: if $\ell(y_1, y_2, z) = 1$, then the worker receives $\phi S(y_2, z)$ if $p(y_2, z) = 1$; and if $\ell(y_1, y_2, z) = 0$ and $p(y_1, z) = 1$, the worker receives $\phi S(y_1, z)$ and in addition she receives $\phi S(y_1, y_2, z)$ if $d(y_1, y_2, z) = 1$.

The asset value of a multiple jobholder is

$$E(y_1, y_2, z) = w(y_1, z) - \omega_1 + w(y_1, y_2, z) - \omega_2 + zg(1 - h(y_1, z) - h(y_1, y_2, z))$$

$$+ \beta \int \Big(\Big(\int (1 - p(y_1', z')) \, dF(y_1'|y_1) \Big) \int \max \{E(y_2', z'), N(z')\} \, dF(y_2'|y_2)$$

$$+ \int \int (p(y_1', z')(E(y_1', z') + d(y_1', y_2', z')(E(y_1', y_2', z')$$

$$- E(y_1', z')))) \, dF(y_2'|y_2) \, dF(y_1'|y_1) \Big) \, dG(z'|z)$$

$$= w(y_1, z) - \omega_1 + w(y_1, y_2, z) - \omega_2 + zg(1 - h(y_1, z) - h(y_1, y_2, z))$$

$$+ \beta \int \Big(N(z') + \Big(\int (1 - p(y_1', z')) \, dF(y_1'|y_1) \Big) \int \max \{E(y_2', z') - N(z')$$

$$, 0\} \, dF(y_2'|y_2) + \int \int (p(y_1', z')(E(y_1', z') - N(z') + d(y_1', y_2', z')(E(y_1', y_2', z')$$

$$- E(y_1', z')))) \, dF(y_2'|y_2) \, dF(y_1'|y_1) \Big) \, dG(z'|z)$$

where we have made use directly of the policy functions $p(y_1, z)$ and $d(y_1, y_2, z)$. Surplus

sharing and making use of $p(y_2, z)$ for the second job yields

$$
E(y_1, y_2, z) = w(y_1, z) - \omega_1 + w(y_1, y_2, z) - \omega_2 + zg(1 - h(y_1, z) - h(y_1, y_2, z))
$$
$$
+ \beta \int \left(N(z') + \left(\int (1 - p(y_1', z')) \, dF(y_1'|y_1) \right) \int \max\{\phi S(y_2', z'), 0\} \, dF(y_2'|y_2) \right.
$$
$$
+ \int \int (p(y_1', z') (\phi S(y_1', z') + d(y_1', y_2', z')
$$
$$
\left. \times \phi S(y_1', y_2', z'))) \, dF(y_2'|y_2) \, dF(y_1'|y_1) \right) dG(z'|z)
$$
$$
= w(y_1, z) - \omega_1 + w(y_1, y_2, z) - \omega_2 + zg(1 - h(y_1, z) - h(y_1, y_2, z))
$$
$$
+ \beta \int \left(N(z') + \left(\int (1 - p(y_1', z')) \, dF(y_1'|y_1) \right) \int \phi p(y_2', z') S(y_2', z') \, dF(y_2'|y_2) \right.
$$
$$
\left. + \int \int (\phi p(y_1', z') (S(y_1', z') + d(y_1', y_2', z') S(y_1', y_2', z'))) \, dF(y_2'|y_2) \, dF(y_1'|y_1) \right) dG(z'|z).
$$

Next, the asset value of employing a single jobholder is

$$
J(y_1, z) = y_1 f(h(y_1, z)) - w(y_1, z) + \beta \int \left(\lambda_1 \int \int ((1 - \ell(y_1', y_2', z')) p(y_1', z') \right.
$$
$$
\times ((1 - d(y_1', y_2', z')) J(y_1', z') + d(y_1', y_2', z') J_1(y_1', y_2', z'))) \, dF_0(y_2') \, dF(y_1'|y_1)
$$
$$
\left. + (1 - \lambda_1) \int \max\{J(y_1', z'), 0\} \, dF(y_1'|y_1) \right) dG(z'|z)
$$
$$
= y_1 f(h(y_1, z)) - w(y_1, z) + \beta \int \left(\lambda_1 \int \int ((1 - \ell(y_1', y_2', z')) p(y_1', z') \right.
$$
$$
\times (J(y_1', z') + d(y_1', y_2', z') (J_1(y_1', y_2', z') - J(y_1', z')))) \, dF_0(y_2') \, dF(y_1'|y_1)
$$
$$
\left. + (1 - \lambda_1) \int \max\{J(y_1', z'), 0\} \, dF(y_1'|y_1) \right) dG(z'|z)
$$

where, again, we have made direct use of the policy functions $p(y_1, z)$ and $d(y_1, y_2, z)$ to simplify notations. Notice that $p(y_1, z)$ multiplies $J_1(y_1, y_2, z)$ in the above as the participation of the primary employer must be ensured. With the surplus sharing rule, we arrive at

$$
J(y_1, z) = y_1 f(h(y_1, z)) - w(y_1, z) + \beta \int \left(\lambda_1 \int \int ((1 - \ell(y_1', y_2', z')) p(y_1', z') \right.
$$
$$
\times ((1 - \phi) S(y_1', z') + d(y_1', y_2', z') (J_1(y_1', y_2', z') - (1 - \phi) S(y_1', z')))) \, dF_0(y_2') \, dF(y_1'|y_1)
$$
$$
\left. + (1 - \lambda_1) \int p(y_1', z') (1 - \phi) S(y_1', z') \, dF(y_1'|y_1) \right) dG(z'|z).
$$

In order to write the asset value of the primary employer, recall that the value in the continuation period depends on $d(y_1, y_2, z)$, the worker's decision to keep the second job, and on the constraint that the job remains viable captured by $p(y_1, z)$. Thus, the asset value of the primary employer is

$$
J_1(y_1, y_2, z) = y_1 f(h(y_1, z)) - w(y_1, z) + \beta \int \int \int (d(y_1', y_2', z') p(y_1', z') J_1(y_1, y_2, z)
$$
$$
+ (1 - d(y_1', y_2', z')) \max\{J(y_1', z'), 0\}) \, dF(y_2'|y_2) \, dF(y_1'|y_1) \, dG(z'|z)
$$

$$= y_1 f\left(h\left(y_1, z\right)\right) - w\left(y_1, z\right) + \beta \int \int p\left(y_1', z'\right) \left(\left(1-\phi\right) S\left(y_1', z'\right) + \int \left(d\left(y_1', y_2', z'\right)\right.\right.$$

$$\left.\left. \times \left(J_1\left(y_1', y_2', z'\right) - \left(1-\phi\right) S\left(y_1', z'\right)\right) dF\left(y_2'|y_2\right)\right) dF\left(y_1'|y_1\right) dG\left(z'|z\right).$$

The last equation uses the surplus sharing rule, and so we arrive at equation (22).

Last, the asset value of being the secondary employer of a multiple jobholder is

$$J_2\left(y_1, y_2, z\right) = y_2 f\left(h\left(y_1, y_2, z\right)\right) - w\left(y_1, y_2, z\right) + \beta \int \left(\int \int \left(d\left(y_1', y_2', z'\right) p\left(y_1', z'\right)\right.\right.$$

$$\left. \times \max\left\{J_2\left(y_1', y_2', z'\right), 0\right\}\right) dF\left(y_2'|y_2\right) dF\left(y_1'|y_1\right) + \left(\int \left(1 - p\left(y_1', z'\right)\right) dF\left(y_1'|y_1\right)\right)$$

$$\left. \times \left(\int \max\left\{J\left(y_2', z'\right), 0\right\} dF\left(y_2'|y_2\right)\right)\right) dG\left(z'|z\right),$$

taking account of the workers' commitment $p\left(y_1, z\right) = \mathbb{1}\left\{J\left(y_1', z'\right) \geq 0\right\}$ towards her primary employer. Therefore we have

$$J_2\left(y_1, y_2, z\right) = y_2 f\left(h\left(y_1, y_2, z\right)\right) - w\left(y_1, y_2, z\right) + \beta \int \left(\int \int \left(d\left(y_1', y_2', z'\right) p\left(y_1', z'\right)\right.\right.$$

$$\left. \times \max\left\{\left(1-\phi\right) S\left(y_1', y_2', z'\right), 0\right\}\right) dF\left(y_2'|y_2\right) dF\left(y_1'|y_1\right) + \left(\int \left(1 - p\left(y_1', z'\right)\right) dF\left(y_1'|y_1\right)\right)$$

$$\left. \times \left(\int \max\left\{\left(1-\phi\right) S\left(y_2', z'\right), 0\right\} dF\left(y_2'|y_2\right)\right)\right) dG\left(z'|z\right)$$

$$= y_2 f\left(h\left(y_1, y_2, z\right)\right) - w\left(y_1, y_2, z\right) + \beta \int \left(\int \int \left(d\left(y_1', y_2', z'\right) p\left(y_1', z'\right)\right.\right.$$

$$\left. \times \left(1-\phi\right) S\left(y_1', y_2', z'\right) dF\left(y_2'|y_2\right) dF\left(y_1'|y_1\right) + \left(\int \left(1 - p\left(y_1', z'\right)\right) dF\left(y_1'|y_1\right)\right)$$

$$\left. \times \left(\int p\left(y_1', z'\right)\left(1-\phi\right) S\left(y_2', z'\right) dF\left(y_2'|y_2\right)\right)\right) dG\left(z'|z\right).$$

Here, we have used the policy functions from Proposition 2 and the surplus sharing equations. In particular, observe that $d\left(y_1', y_2', z'\right) \max\left\{J_2\left(y_1', y_2', z'\right), 0\right\} = d\left(y_1', y_2', z'\right) J_2\left(y_1', y_2', z'\right)$ since $d\left(y_1', y_2', z'\right) = \mathbb{1}\left\{S\left(y_1', y_2', z'\right) \geq 0\right\} = \mathbb{1}\left\{J_2\left(y_1', y_2', z'\right) \geq 0\right\}$.

To complete the derivation, add up the last equations we have obtained for $E\left(y_1, z\right)$ and $J\left(y_1, z\right)$ and subtract $N\left(z\right)$ in order to arrive at Equation (17) ($S\left(y_1, z\right)$). Similarly, add up the last equations we have obtained for $E\left(y_1, y_2, z\right)$ and $J_2\left(y_1, y_2, z\right)$ and subtract $E\left(y_1, z\right) = \phi S\left(y_1, z\right) + N\left(z\right)$ to arrive at Equation (20) ($S\left(y_1, y_2, z\right)$). Finally, to recover the wage functions, rearrange the last equation we have obtained for $J\left(y_1, z\right)$ and $J_2\left(y_1, y_2, z\right)$ to compute $w\left(y_1, z\right)$ and $w\left(y_1, y_2, z\right)$. In these calculations, use $J\left(y_1, z\right) = \left(1-\phi\right) S\left(y_1, z\right)$ and $J_2\left(y_1, y_2, z\right) = \left(1-\phi\right) S\left(y_1, y_2, z\right)$ on the left-hand side of each equation.

A.4   Proof of Proposition 3

Since the function $f\left(.\right)$ in (28) is not differentiable everywhere, we cannot apply the second part of Proposition 1. However, the first part of the Proposition, i.e. surplus sharing through wages,

remains valid. For primary jobs (the logic is analogous for second jobs) we have $E(y_1, z) - N(z) = \phi S(y_1, z)$ and $J(y_1, z) = (1 - \phi) S(y_1, z)$. Substituting into the Nash product, we obtain

$$(E(y_1, z) - N(z))^\phi J(y_1, z)^{1-\phi} = \phi^\phi (1-\phi)^{1-\phi} S(y_1, z).$$

Thus, agents choose hours worked to maximize the joint surplus. Given rebargaining every period, this amounts to maximizing the sum of market and home productions, $yf(h) + zg(1-h)$.

The specific functional forms for $g(1-h)$ and $f(h)$ in respectively (27) and (28) give rise to the following possibilities, illustrated in Figure A1:
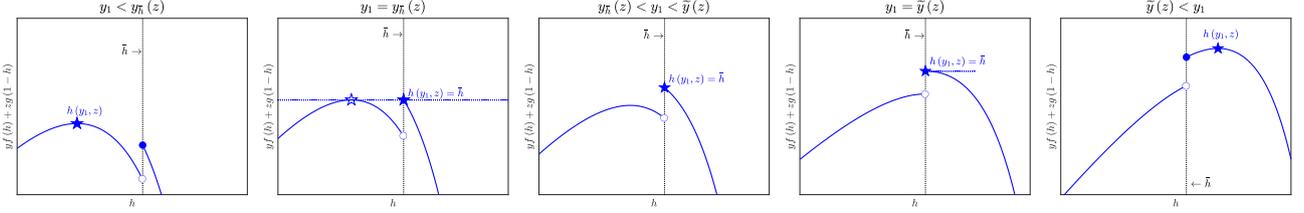

Figure A1: Illustration of Proposition 3

First, there can be an interior maximum of $yf(h) + zg(1-h)$ attained at some h to the left of $\bar{h}$. Using the first-order condition, we have: $h(y_1, z) = 1 - \left(\frac{z}{(1-\psi)y_1}\right)^\gamma$. As y increases, $yf(h) + zg(1-h)$ shifts up, and there is a value $y_{\bar{h}}(z)$ such that the interior solution to the left of $\bar{h}$ yields the same maximum value as $yf(\bar{h}) + zg(1-\bar{h})$. Thus, $y_{\bar{h}}(z)$ is defined by

$$y_{\bar{h}}(z) f(h(y_{\bar{h}}(z), z)) + zg(1 - h(y_{\bar{h}}(z), z)) = y_{\bar{h}}(z) f(\bar{h}) + zg(1 - \bar{h}), \tag{34}$$

with $h(y_{\bar{h}}(z), z) = 1 - \left(\frac{z}{(1-\psi)y_{\bar{h}}(z)}\right)^\gamma$. For values of y_1 greater than $y_{\bar{h}}(z)$ but strictly below some $\widetilde{y}(z)$, there is a local maximum in the $[0, \bar{h})$ interval, but the global maximum is given by $h(y_1, z) = \bar{h}$. At $y_1 = y_{\bar{h}}(z)$, the maximum attained at $h = \bar{h}$ satisfies the first-order condition. As a result, we have:

$$\widetilde{y}(z) = \frac{z(1-\bar{h})^{-\frac{1}{\gamma}}}{1-\psi}. \tag{35}$$

For values of y_1 above $\widetilde{y}(z)$, the interior solution to the right of $\bar{h}$ is the global maximum.

We define likewise a cutoff value for second jobs $y_{\bar{h}}(y_1, z)$ which satisfies

$$y_{\bar{h}}(y_1, z) f(h(y_1, y_{\bar{h}}(y_1, z), z)) + zg(1 - h(y_1, z) - h(y_1, y_{\bar{h}}(y_1, z), z))$$
$$= y_{\bar{h}}(y_1, z) f(\bar{h}) + zg(1 - h(y_1, z) - \bar{h}), \tag{36}$$

with $h(y_1, y_{\bar{h}}(y_1, z), z) = 1 - h(y_1, z) - \left(\frac{z}{(1-\psi)y_{\bar{h}}(y_1, z)}\right)^\gamma$. Observe that $1 - h(y_1, z)$ is the upper bound on the hours that can be allocated to the second job. The upper threshold $\widetilde{y}(y_1, z)$ that is the analogous of (35) is given by

$$\widetilde{y}(y_1, z) = \frac{z(1 - h(y_1, z) - \bar{h})^{-\frac{1}{\gamma}}}{1-\psi}. \tag{37}$$

This completes the proof of Prop. 3 (with Equations (34)–(37) mentioned in the Proposition).

B Data appendix

B.1 CPS worker flows data

The data we use to inform the calibration of the model come from the monthly files of the Current Population Survey (CPS). The CPS is a survey of households administered by the U.S. Census Bureau under the auspices of the U.S. Bureau of Labor Statistics. Since January 1994, the CPS has been collecting information which allow to identify multiple jobholders. The survey asks respondents about the number of jobs held during the reference week, whether they usually receive a wage or salary from the primary job, and collects information on hours worked for up to two jobs. Starting in 1994, the CPS also measures whether individuals change employers from one month to the next. We use data from January 1994 through March 2020.

CPS respondents are interviewed for four consecutive months, are rotated out of the survey for eight months, and are included in the survey again for four consecutive months. As a result, in each monthly file of the CPS, up to three-quarters of respondents can be longitudinally matched across two consecutive months of the survey.

Throughout the analysis, we focus on workers aged 25 to 54 years old with some College or higher education. We exclude workers who report being self-employed in any spell of employment.

Empirical framework. We use a stock-flow framework to compute transitions in and out of multiple jobholding. In each period t, individuals are classified into one of the following states: multiple jobholding with a full-time primary job (F_M), multiple jobholding with a part-time primary job (P_M), single jobholding with a full-time job (F_S), single jobholding with a part-time job (P_S), and nonemployment (N). We let the vector $\boldsymbol{s}_t$ contain the number of individuals (stocks) in each of these states:

$$\boldsymbol{s}_t = [\ \underbrace{F_M \quad P_M}_{M} \ \underbrace{F_S \quad P_S}_{S} \quad N\ ]'_t,$$

where $M = F_M + P_M$ (resp. $S = F_S + P_S$) is the number of multiple jobholders (resp. single jobholders) in period t. As is standard, the evolution of $\boldsymbol{s}_t$ is described by means of a discrete-time, first-order Markov chain:

$$\boldsymbol{s}_t = \boldsymbol{\Pi}_t \boldsymbol{s}_{t-1}.$$

In this equation, $\boldsymbol{\Pi}_t$ is the stochastic matrix of transition probabilities across labor market states i and j. Each of these transition probabilities is measured by the gross flow of workers from state i to state j at time t divided by the stock of workers in state i at time $t-1$. We clear transition probabilities from several measurement issues: we adjust them to control for systematic seasonal variations, margin error discrepancies and time-aggregation bias (see Lalé [2016] for details about these adjustments). The margin error adjustment is useful to address the fact that we exclude employed workers if they report being self-employed.

B.2 CPS work schedule supplements data

Fact 4 described in Section 2 is based on data from the CPS supplements on Work Schedules and Work from Home. These data are available in May for the years 1997, 2001 and 2004. As the name suggests, the supplements aim at gathering additional information relevant to our understanding of an individual's work schedule. For multiple jobholders, the supplements record the occupation of employment of the second job. We merge this information with

the occupational classification provided by IPUMS-CPS (https://cps.ipums.org/cps/) to harmonize the occupational classification and pool together data from the three survey waves. The final sample contains about 3,000 multiple jobholders.[32]

Descriptive statistics. We first compare the occupations of the primary and second jobs of multiple jobholders. Table B1 reports that there are about 30 percent of multiple jobholders whose second job's occupation is the same as that of the primary job at the 2-digit level. This rate is similar to Table 2 in Paxson and Sicherman [1996], with some differences that could be due to sample selection.[33]

Table B1: Multiple jobholding, same primary and second job's occupations

		Occupation		
		1-digit	2-digit	3-digit
Share with same primary and second job occupations	**M**	46.6	30.1	19.6
	W	56.2	29.8	17.6

Notes: The table reports the share of multiple jobholders whose second job's occupation is the same as that of their primary job. The 1-digit, 2-digit, 3-digit levels of the occupational classification contain respectively 6, 25, and 450 occupations. M and W denote respectively men and women. Data come from the May "Work Schedule and Work at Home" supplements of the Current Population Survey for individuals aged 25 to 54 with some College or higher education.

We then ask a slightly different question, namely the extent to which a worker's primary job occupation is predictive of the occupation of her second job. The first column of Table B2 reports that 14 percent (11 percent) of male (female) multiple jobholders have their main job in an occupation where 100% of second jobs belong to the same (3-digit) occupation. To be clear, it could be that in primary job occupation A, all second jobs are also in occupation A; and that in primary job occupation B, all second jobs are in occupation C. In both instances the primary job occupation is fully predictive of the second job occupation. To use a less stringent criterion, we then consider instances where one occupation accounts for at least $X\%$ of all second jobs, again conditional on the primary job occupation. By construction, as X decreases towards 0, this statistics accounts for an increasingly larger share of multiple jobholding. The decrease, however, is not very steep. For instance, 31 percent (21 percent) of male (female) multiple jobholders have their main job in an occupation where more than 60 percent of second jobs belong to the same (3-digit) occupation. As will be explained below, these figures indicate that second job occupations are highly concentrated within multiple jobholders' primary occupation of employment.

Table B2: Share of second jobs partially accounted for by only one occupation

		% 2nd jobs in the same occ.				
		100	≥ 80	≥ 60	≥ 40	≥ 20
Share of all multiple jobholders	**M**	13.8	24.7	31.3	55.5	91.3
	W	11.2	12.4	20.7	49.5	82.7

Notes: The table reports the share of multiple jobholders who are employed in a primary occupation where at least $X\%$ of all second jobs are accounted for by only one occupation. Occupations are defined at the 3-digit level. M and W denote respectively men and women. Data come from the May "Work Schedule and Work at Home" supplements of the Current Population Survey for individuals aged 25 to 54 with some College or higher education.

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[32]We use the CPS supplements weights to calculate the data moments reported in this section.

[33]Table 4 in Paxson and Sicherman [1996] shows rates of same primary and second job occupation at the 2-digit level at 22 and 28 percent for respectively men and women. We restrict the sample to 25-54 years old workers with some College or higher education. Thus, our sample has a greater representation of occupations with a high rate of same primary and second job occupations (medical professionals, accountants, and lawyers).

There is yet another way to think about the predictive power of the primary job occupation vis-à-vis the second job. How many multiple jobholders have their primary job in an occupation where at most N occupations account for 100% of all second jobs? By construction, as N increases towards 450 (the total number of 3-digit occupations), this statistics accounts for an increasingly larger share of multiple jobholding. Table B3 shows, however, that $N = 5$ is enough to describe the occupations of most (58 percent of men and 45 percent of women) multiple jobholders. This means that these workers are in occupations that connect to only few (at most five) occupations when it comes to having a second job – equivalently, that second job occupations are highly concentrated within multiple jobholders' primary occupation of employment.

Table B3: Share of second jobs fully accounted for by a few occupations

		100% of 2nd jobs within at most				
		1 occ.	2 occ.	3. occ	4 occ.	5 occ.
Share of all multiple jobholders	**M**	13.8	26.7	42.6	53.8	58.1
	W	11.2	19.6	31.6	40.3	44.6

Notes: The table reports the share of multiple jobholders who are employed in a primary occupation where 100% of all second jobs are accounted for by at most N occupations. Occupations are defined at the 3-digit level. M and W denote respectively men and women. Data come from the May "Work Schedule and Work at Home" supplements of the Current Population Survey for individuals aged 25 to 54 with some College or higher education.

Why are these statistics an indication of occupational concentration of second jobs? What is the benchmark to appreciate the data moments reported in Tables B2 and B3? To answer these questions, we compare those statistics with results that we obtain by looking at occupational mobility among single jobholders. We check whether a worker's initial occupation is highly predictive of her future occupation, as opposed to occupational changes being vastly dispersed across occupations conditional on a worker's initial occupation. To do so, we longitudinally match CPS respondents across two consecutive months of interview, compare their 3-digit occupations between the two months, and we pool data from 1994 through 2020 to obtain a large sample of occupational movers. The results are overwhelming. Consider for instance Table B3. When we apply the same statistics to occupational mobility among single jobholders, we find that there are *no* workers employed in an occupation where at most 5 'destination occupations' account for all occupational switches. For the analogue of Table B2, we obtain 0s in the first 3 columns. In the 4th column, we find that 3 percent of workers are employed in jobs where a single 'destination' explains at least 40 percent of mobility within the occupation. The corresponding figures for multiple jobholding in Table B2 are 56 percent for men and 50 percent for women. Together, these findings support the conclusion that the primary occupation of a multiple jobholder strongly conditions the set of occupations in which she holds her second job.

C   Additional results

Table C1 is the analogue of Table 2 in the main text. The table presents the outcomes of different calibrations that target values of the Frisch elasticity of 0.30 and 0.60. The other calibration targets are the same as in the baseline calibration. The model fit as measured in the last set of columns of Table C1 remains very similar. To illustrate graphically the difference between the different calibrations (i.e., $\mathcal{F} = 0.30$, $\mathcal{F} = 0.45$, $\mathcal{F} = 0.60$), Figure C1 plots the distribution of home productivity, z, of each parameterization. A larger value of the Frisch

Table C1: Internally calibrated parameters: Lower ($\mathcal{F} = 0.30$) and higher ($\mathcal{F} = 0.60$) Frisch elasticity of labor supply

Parameter	Description		Value $\mathcal{F}=0.30$	Value $\mathcal{F}=0.60$	Targeted moment		Data $\mathcal{F}=0.30$	Data $\mathcal{F}=0.60$	Model $\mathcal{F}=0.30$	Model $\mathcal{F}=0.60$
γ	Curvature of $g(1-h)$	M	0.234	0.467	Frisch elasticity of labor supply*	M	0.30	0.60	0.30	0.60
		W	0.187	0.374		W	0.30	0.60	0.30	0.60
μ_z	Home prod., uncond. mean	M	0.124	0.374	Part-time empl. share	M	4.92		4.93	4.87
		W	0.093	0.378		W	17.6		17.9	17.8
ρ_z	Home prod., persistence	M	0.799	0.833	Full- to part-time trans. prob.	M	1.29		1.27	1.18
		W	0.982	0.995		W	3.13		3.25	3.39
σ_z	Home prod., standard dev.	M	0.066	0.146	Average hours per worker	M	43.8		44.2	44.0
		W	0.025	0.085		W	38.4		39.0	39.2
ψ	Prod. gap at $\bar{h}$ hours	M	0.033	0.044	Share bunching at full-time hours	M	44.2		44.1	44.1
		W	0.011	0.024		W	45.1		45.8	45.8
κ	Vacancy posting cost	M	0.071	0.071	Expected vac. cost / qrtly earnings*	M	14.0		14.1	14.0
		W	0.060	0.062		W	14.0		14.0	14.1
σ_ε	Match prod., standard dev.	M	0.054	0.057	Empl. separation rate	M	1.79		1.75	1.82
		W	0.106	0.120		W	2.63		2.59	2.74
δ	Separation shock	M	0.005	0.004	Share exogenous empl. separation*	M	25.0		25.0	25.0
		W	0.007	0.007		W	25.0		25.0	25.0
s_e	On-the-job search efficiency	M	0.317	0.313	Job-to-job transition rate	M	1.71		1.72	1.71
		W	0.302	0.307		W	1.75		1.76	1.75
ω_1	Cost of working job 1	M	0.195	0.098	Employment rate	M	95.0		95.1	94.9
		W	0.196	0.082		W	93.5		93.6	93.2
ω_2	Cost of working job 2	M	0.038	0.041	Multiple jobholding share	M	6.57		6.57	6.61
		W	0.072	0.083		W	6.62		6.75	6.60

Notes: The table describes the model parameters (left panel) that provide the best fit to the data (right panel). M and W denote model and data moments for respectively men and women; $\mathcal{F}$ denotes the targeted Frisch elasticity of labor supply. The model period is set to be one month. The data moments except those marked with an asterisk are based data from the Current Population Survey for individuals aged 25 to 54 with some College or higher education; those marked with an asterisk are taken from the literature. All moments except the Frisch elasticity of labor supply and average hours per worker are expressed in percent.
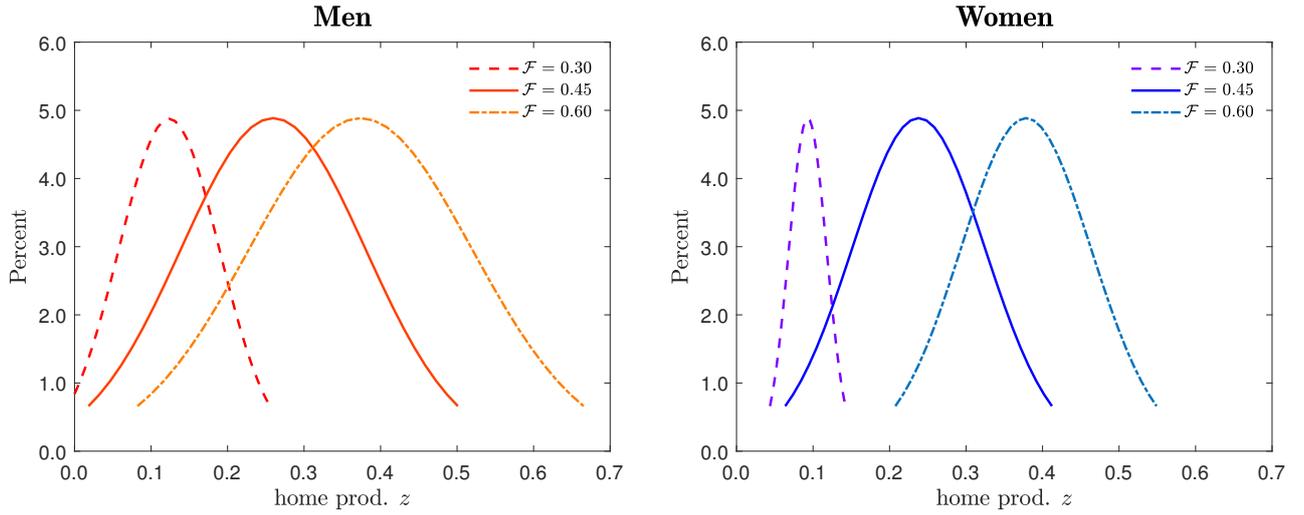
Figure C1: Distribution of home productivity z

Notes: The panels in this figure plot, for men and women, the cross-sectional distribution of worker's home productivity, z. In each panel, the lines refer to models calibrated to match three different values (0.30, 0.45, 0.60) of the Frisch elasticity of labor supply, $\mathcal{F}$.

elasticity shifts the mean and the dispersion of those distributions to the right. It also implies more persistence of the stochastic process.

To illustrate further the workings of the model, in Figure C2 we plot a reservation threshold called $\tilde{y}_d(y_1, z)$ that plays a key role in the model's equilibrium. $\tilde{y}_d(y_1, z)$ is defined by: $E(y_1, \tilde{y}_d(y_1, z), z) = E(y_1, z)$, i.e. it is the lowest value of match productivity of the second job y_2 for the worker to accept it. As can be seen $\tilde{y}_d(y_1, z)$ increases with both match productivity of the first job y_1 and home productivity z. When match productivity of the first job is higher, the worker has fewer hours to devote to the second job, which makes her more selective about the second job. The worker is also more selective when she is less willing to add more hours. This occurs when her own idiosyncratic home productivity is higher. Note that $\tilde{y}_d(y_1, z)$ describes the worker's decision when she is not switching to the outside employer, i.e. when the match productivity draw at the outside employer is $y_2 \leq y_1$.
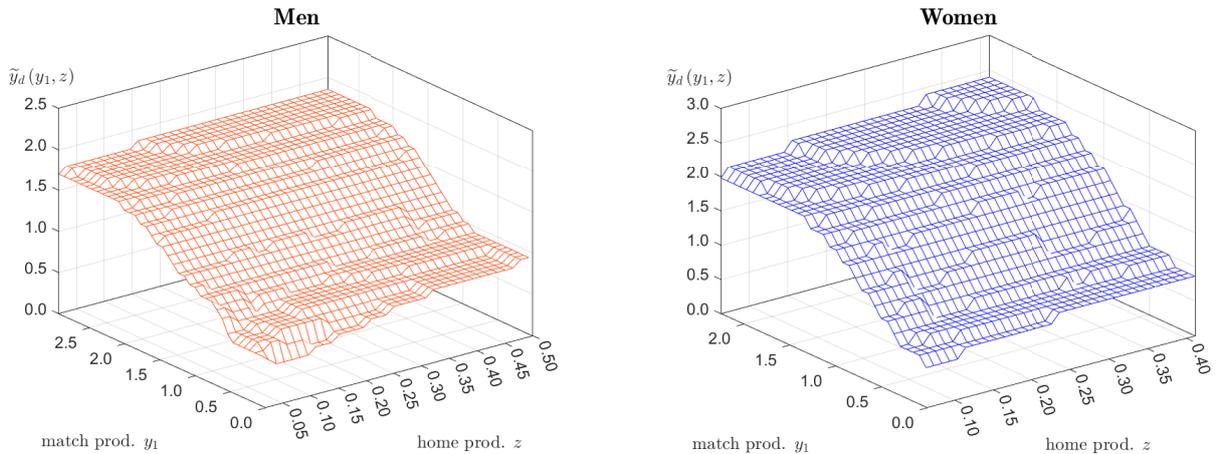

Figure C2: Reservation thresholds for multiple jobholding

Notes: The panels in this figure plot, for men and women, the reservation thresholds $\tilde{y}_d(y_1, z)$ that define the decision to take on a second job. $\tilde{y}_d(y_1, z)$ depends on match productivity of the first job, y_1, and the workers' home productivity, z. The underlying parameter values are those matching a Frisch elasticity of labor supply of 0.45.

Table C2 describes the calibrated parameter values of the model without multiple jobholding (with $\mathcal{F} = 0.45$ as a target for the Frisch elasticity). In Table 6 of the text, we summarized the main differences between the parameter values of this model and those of the full model with multiple jobholding. It is useful to note that in this version too, the model achieves a very good fit to the targeted data moments (last column of Table C2).

Table C2: Internally calibrated parameters: Model without multiple jobholding

Parameter	Description		Value	Targeted moment		Data	Model
γ	Curvature of $g(1-h)$	M	0.350	Frisch elasticity of labor supply*	M	0.45	0.45
		W	0.281		W	0.45	0.45
μ_z	Home prod., uncond. mean	M	0.249	Part-time empl. share	M	4.92	5.00
		W	0.228		W	17.6	17.3
ρ_z	Home prod., persistence	M	0.825	Full- to part-time trans. prob.	M	1.29	1.31
		W	0.946		W	3.13	3.10
σ_z	Home prod., standard dev.	M	0.120	Average hours per worker	M	43.8	44.2
		W	0.076		W	38.4	39.7
ψ	Prod. gap at $\bar{h}$ hours	M	0.040	Share bunching at full-time hours	M	44.2	44.2
		W	0.022		W	45.1	45.9
κ	Vacancy posting cost	M	0.075	Expected vac. cost / qrtly earnings*	M	14.0	14.0
		W	0.063		W	14.0	14.1
σ_ε	Match prod., standard dev.	M	0.043	Empl. separation rate	M	1.79	1.78
		W	0.101		W	2.63	2.63
δ	Separation shock	M	0.005	Share exogenous empl. separation*	M	25.0	25.0
		W	0.007		W	25.0	25.0
s_e	On-the-job search efficiency	M	0.314	Job-to-job transition rate	M	1.71	1.73
		W	0.347		W	1.75	1.76
ω_1	Cost of working job 1	M	0.134	Employment rate	M	95.0	94.9
		W	0.119		W	93.5	93.5

Notes: The table describes the model parameters (left panel) that provide the best fit to the data (right panel). M and W denote model and data moments for respectively men and women. The model period is set to be one month. The data moments except those marked with an asterisk are based data from the Current Population Survey for individuals aged 25 to 54 with some College or higher education; those marked with an asterisk are taken from the literature. All moments except the Frisch elasticity of labor supply and average hours per worker are expressed in percent.