CIRANO
Centre interuniversitaire de recherche
en analyse des organisations

Série Scientifique
Scientific Series

95s-40

Vertical Integration, Foreclosure and Profits in the Presence of Double Marginalisation

Gérard Gaudet, Ngo Van Long

Montréal
septembre 1995

CIRANO

Le CIRANO est une corporation privée à but non lucratif constituée en vertu de la Loi des compagnies du Québec. Le financement de son infrastructure et de ses activités de recherche provient des cotisations de ses organisations-membres, d'une subvention d'infrastructure du ministère de l'Industrie, du Commerce, de la Science et de la Technologie, de même que des subventions et mandats obtenus par ses équipes de recherche. La *Série Scientifique* est la réalisation d'une des missions que s'est données le CIRANO, soit de développer l'analyse scientifique des organisations et des comportements stratégiques.

CIRANO is a private non-profit organization incorporated under the Québec Companies Act. Its infrastructure and research activities are funded through fees paid by member organizations, an infrastructure grant from the Ministère de l'Industrie, du Commerce, de la Science et de la Technologie, and grants and research mandates obtained by its research teams. The Scientific Series *fulfils one of the missions of CIRANO: to develop the scientific analysis of organizations and strategic behaviour.*

Les organisations-partenaires / The Partner Organizations

- Ministère de l'Industrie, du Commerce, de la Science et de la Technologie.
- École des Hautes Études Commerciales.
- École Polytechnique.
- Université de Montréal.
- Université Laval.
- McGill University.
- Université du Québec à Montréal.
- Bell Québec.
- La Caisse de dépôt et de placement du Québec.
- Hydro-Québec.
- Fédération des caisses populaires de Montréal et de l'Ouest-du-Québec.
- Téléglobe Canada.
- Société d'électrolyse et de chimie Alcan Ltée.
- Avenor.
- Service de développement économique de la ville de Montréal.
- Raymond, Chabot, Martin, Paré

ISSN 1198-8177

Vertical Integration, Foreclosure and Profits in the Presence of Double Marginalisation*

Gérard Gaudet†, *Ngo Van Long*‡

Résumé / Abstract

La rentabilité de l'intégration verticale entre une firme à l'amont et une firme à l'aval dépend de la manière de poser la question, du nombre de firmes et du type d'interaction entre les firmes intégrées et les firmes non-intégrées. Si l'on n'impose aucune contrainte sur les transactions entre les firmes intégrées et les firmes non-intégrées, alors les premières peuvent continuer à acheter l'input produit par les firmes non-intégrées à l'amont afin de hausser les coûts des firmes non-intégrées à l'aval. On montre que, même dans le cas où les firmes sont identiques, il y a des équilibres asymétriques dans le jeu d'intégration.

Whether vertical integration between a downstrean oligopolist and an upstream oligopolist is profitable for an integrated pair of firms is shown to depend on how one formulates the questions, on the number of firms in each oligopoly and on the type of interaction which is assumed between firms that are integrated and firms that are not. In particular, it is shown that if no restriction is put on trade between integrated and non integrated firms, integrated firms may continue to purchase inputs from the non integrated upstream firms, with the goal of raising their downstream rival's costs. Furthermore, even though firms are identical, asymmetric equilibria, where integrated and non integrated firms coexist, may actually arise as an outcome of the integration game.

Mots clé : Intégration verticale, jeu d'intégration, équilibre asymétrique

Key words : Integration vertical, integration game, asymmetric equilibria

---

* We are indebted to Andrew Daughety, Jennifer Reinganum and Antoine Soubeyran for helpful discussions when we began work on this paper. The contents of an anonymous co-editor and an anonymous referee have force us to clarify a number of issues and have thus helped improve the paper. We also wish to thank for their comments Jacques Crémer and Roger Ware as well as seminar participants at Université Laval, Université de Montréal, Flinders University, Université d'Aix-Marseille II, Queen's University, University of Melbourne, and the Canadian Bureau of Competition Policy. Financial support from the Fonds FCAR du Gouvernement du Québec and the Conseil de Recherche en Science Humaine du Canada is gratefully acknowledged.

† Université Laval & CIRANO

‡ McGill University & CIRANO

1. Introduction

It is well known that the vertical integration of a downstream monopolist and an upstream monopolist is profitable, in the sense that the profit of the integrated entity will exceed the combined pre-integration profits. The reason for this is that the presence of double marginalisation results in the consumer being charged a price that exceeds the monopoly price which would be chosen by the integrated firm. We show in this paper that whether the result still holds when the vertical integration occurs between a downstream oligopolist and an upstream oligopolist depends on how the question is formulated, as well as on the number of firms in the oligopolies and the type of interaction allowed between those that are integrated and those that are not. We show this using a simple linear demand example, where firms compete in quantities at both stages of production after having taken their decisions whether to integrate or not.

In the case of successive oligopolists, there are in fact various pertinent ways of addressing the question. A natural way consists in asking whether vertical integration constitutes a dominant strategy, in the sense that it results in greater consolidated profits for the pair of firms that integrate irrespective of the integration decision of the other firms. A second way consists in asking whether a situation where all firms vertically integrate is superior to one where no firm does, in the sense that each is left better off. This second question of course raises the further question of whether such a situation can constitute an equilibrium. In other words, is the total profit earned by an integrated firm from its upstream and downstream divisions greater (or at least not smaller) than what the two divisions would jointly earn if they operated as non integrated entities, given that the rest of the industry is integrated[1].

[1]It is often presumed that the results for successive monopolists carry over to successive oligopolists. For instance, Perry (1989), after having discussed the fact that vertical integration of successive monopolists reduces the final price, thereby increasing joint profits and consumer welfare, states that "The analysis and results would obviously be identical for forward integration by upstream oligopolists into a downstream oligopoly." (see his footnote 13, page 199). Concerning profits this turns out to be true only in some cases and to depend on the way the question is formulated. A number of authors have touched upon related questions. Bonanno and Vickers (1988), Lin(1988) and Nakache and Soubeyran (1989a) find that when differentiated

To analyse those questions, we consider an integration game where in a first stage firms take the vertical merger decision based on the anticipated equilibrium profits that occur in the subsequent production stages. This of course requires that we solve for situations where integrated and non integrated firms coexist, whether such situations will actually occur in equilibrium or not. This in turn requires some assumption about the interaction between integrated and non integrated firms. We will generally assume that an integrated firm can freely purchase from or sell to the non integrated firms. A striking result is that if some firms are integrated and some not, an integrated firm may choose to buy some of its inputs from the non integrated upstream sector at a market price which exceeds its own upstream marginal cost of production. The reason is that this drives up the downstream rivals' costs, thus reducing downstream competition. Depending on the relative importance of integrated firms in the downstream market, this reduction in downstream competition may more than compensate for the higher input cost.

This result is obviously important when studying situations where, for whatever reasons, integrated and non integrated firms coexist. Often, the coexistence of integrated and non integrated firms is simply *assumed* in analysis of equilibrium foreclosure (for example Salinger(1988) and Krattenkramer and Salop (1986)). We show here that such situations may in fact arise as equilibria of the integration game, even though firms are identical at the outset. Furthermore, those asymmetric subgame perfect equilibria may feature the type of raising rivals' costs strategy just described. It also turns out, however, that the mere possibility of adopting such strategies out of (subgame perfect) equilibrium is important in determining the type and characteristics of the equilibria that may arise out of the in-

goods producers and retailers compete in price, the equilibrium may dictate vertical separation. Under quantity competition, Nakache and Soubeyran (1989b) also observe that in the duopoly case integration can be a dominant strategy. Lin (1988, footnote 7, page 254) alludes to, without explaining, the possibility of a prisoner's dilemma arising in a special case. Greenhut and Ohta (1979, footnote 9, page 140) implicitly seem to envisage a similar situation, without explicitly recognizing it. Salinger (1988, pages 353–354), in a model where vertically integrated and non integrated producers are exogenously made to coexist, finds that integration does not necessarily increase the joint profits of the integrated firms.

tegration game, whether or not such partially integrated situations can actually occur as equilibria.

Our analysis clearly establishes that the answers to the questions posed at the outset depend crucially on the number of firms at each stage of production. To illustrate, we find that with successive duopolists it is always profitable for a pair of downstream and upstream firms to vertically integrate unilaterally, no matter what the other pair of firms does. However when both integrate, which turns out to be the only equilibrium to the integration game in this case, the consolidated profit of each is less than it would be if both chose not to integrate. Hence we are in the presence of a prisoner's dilemma.[2] If we simultaneously increase the number of upstream and downstream firms, then, when the number of firms gets large enough (five of more in our linear example), to unilaterally integrate is not a dominant strategy any more — whether it pays to integrate depends on how many pairs it expects will integrate — and there are now two equilibria to the integration game, one where everyone integrates and one where no one integrates. What was a prisoner's dilemma becomes a coordination problem. Still other possibilities arise in the more general case where the number of upstream and dowmstream firms differ. For instance, we find that when there are two upstream firms and more than two downstream firms, if the two upstream firms each integrate a downstream firm, they always make less than the joint profit of the same pair with no one integrated. However, depending on the number of downstream firms, to integrate may or may not be a dominant strategy; the only equilibrium may be for each to integrate or for none to integrate; the only equilibria may be for one upstream firm to be integrated and the other not. On the other hand, if there were two downstream firms and more than two upstream firms, the only equilibrium is for the two downstream firms to be integrated and each integrated firm is then better off than if none were integrated.

[2]The reason is that in the case of successive oligopolists, vertical integration will, by reducing the cost of the input into the downstream production process, increase the degree of competition in the downstream market, thus mitigating the gains from eliminating the double marginalisation.

Since it is often explicitly assumed in the literature on vertical integration that an integrated firm does not directly interact with independent upstream or downstream producers, we also briefly consider the consequences of imposing complete foreclosure, whereby no purchases or sales can occur between integrated and non integrated firms. We show that modelling the problem in this way does indeed have important consequences for the type of equilibria that may arise in the integration game, even in situations where asymmetric equilibria — with some firms integrated and some not — cannot occur. For instance, if the number of upstream and downstream firms is initially the same, there may generally coexist an equilibrium where all firms are integrated and an equilibrium where none are. If however complete foreclosure were imposed, there would exist no equilibrium where no firm integrates. To take another example, we find that when there are two upstream firms and more than two downstream firms, if the two upstream firms each integrate a downstream firm they generally make less than the joint profit of the same pair with no one integrated, but not if foreclosure is imposed; full integration and non integration cannot generally coexist as equilibria, but can when foreclosure is imposed; asymmetric equilibria, where only one of the two upstream firms integrates, can never occur when foreclosure is imposed, whereas they can otherwise.

This paper is related in some respects to that of Salinger (1988). He also considers successive oligopolies with competition in quantities at both stages. He however imposes exogenously the coexistence of integrated and non integrated firms and does not study the integration game as such. More importantly perhaps, his explicit assumptions are such that the integrated firms choose not to trade in any way with the non integrated firms (complete foreclosure). Insightful analysis of equilibrium foreclosure have recently been proposed by Ordover, Saloner and Salop (1990) and Hart and Tirole (1990). However both of those papers focus exclusively on the case where upstream firms compete in price[3] thereby eliminating

[3]Along with the usual Bertrand competition, Hart and Tirole also allow for a more uneven distribution of bargaining power between the upstream and downstream firms. But in all cases the double marginalisation

the double marginalisation effect, which is central to the question posed at the outset. They also implicitly assume that when integrated and non integrated firms coexist, the trade of the integrated firms with the non integrated one is restricted to non negative net sales of inputs. As discussed above and as we will show below, allowing net sales of the upstream product by the integrated firms to be negative (i.e., purchases of input), as we do, turns out to be not inconsequential.

We will neglect in this paper all welfare or direct policy discussions. Our aim is simply to provide some new insight as to what determines the private profitability of vertical integration in the presence of double marginalisation.

In the next section we discuss the precise structure we are assuming for the integration game. This is followed in section 3 by an analysis of the equilibria of the production sub-games. Section 4 deals with the integration game as such. We end with a brief conclusion in section 5.

2. The structure of the game

The vertical integration problem is modelled as a two stage game: an integration stage, followed by a production stage. In a first stage, the existing firms *simultaneously* decide whether to vertically integrate or not. They take their decisions based on the anticipated equilibrium profits resulting from the second stage. In the second stage, the firms decide how much to produce, taking as given the industry structure which results from the first stage. We thus seek subgame perfect equilibria of the integration game.

The production decision itself has two substages, reflecting the vertically related structure of production. In the first substage, the vertically integrated firms and the non integrated upstream producers simultaneously decide on the quantity of the upstream good to produce.

motive for integration is neutralised by broadening the type of contractual arrangements (e.g., two-parts tariffs.)

In doing so, they face the derived demand for the upstream product anticipated from the dowstream equilibrium decisions of the non integrated downstream firms. In the downstream substage, the integrated firms and the non integrated downstream firms compete in the quantity produced of the final good, taking as given the price of the upstream good they use as input and the consumer demand for the final good. The detailed modelling of each of these substages is spelled out in the next section.

Although quite frequently encountered in the vertical integration literature, this sequential Cournot specification of the production game is somewhat arbitrary. When the number of upstream and downstream firms is small, one might want to view the determination of the price of the upstream product as being the result of a multilateral bargaining game between upstream and downstrean firms. Our specification amounts to drastically simplifying this multilateral bargaining process by implicitly assuming that the upstream firms are in a position to make take it or leave it offers. There are obviously many alternatives to this one-sided bargaining specification. We believe however that this is an attractive and tractable way of highlighting the double marginalisation effect inherent to the oligopolistic vertical structure.

The role of double marginalisation is the focus of the paper. It is true that non-linear prices may take care of the double margin in some situations and hence remove incentives to vertically integrate. This is clearly the case in a successive monopoly context. However it remains to be shown that such contracts are always implementable in a more general successive oligopoly context and, when implementable, that the resulting equilibrium non-linear contracts would in fact completely eliminate the double margin. Thus we believe that double marginalisation remains an important consideration in the analysis of vertical integration.

3. The equilibria of the production subgames

Consider a situation where, in the absence of vertical integration, $n_d \geq 2$ downstream firms transform one for one the homogeneous output of $n_u \geq 2$ upstream firms into a homogeneous final product which they then sell to the consumers. Assume for simplicity that there are no costs to produce the upstream good nor to transform it into the final product. The downstream firms pay a price w for their input, which they take as given, and sell the final product at price p.

The inverse demand for the final product is given by $p = 1 - Y$, where $Y = \sum_{i=1}^{n_d} y_i$ is the total downstream output, y_i being the individual output. Hence downstream firms each receive profits of $[1 - Y - w]y_i$, $i = 1, 2, \ldots, n_d$. They compete in quantities, so that the downstream market is an n_d-firm Cournot oligopoly with identical marginal costs of w. It is a simple matter to verify that equilibrium in this downstream market requires that each downstream firm produce $y_i = (1 - w)/(n_d + 1)$, for a total of $Y = n_d(1 - w)/(n_d + 1)$.

Let $X = \sum_{i=1}^{n_u} x_i$ denote the total upstream production, with x_i representing the production of upstream firm i. Since downstream firms transform the upstream product one for one, we must have $Y = X$. The derived inverse demand faced by the upstream industry is therefore $w = 1 - (n_d + 1)X/n_d$ and hence upstream firm i receives a profit of $[1 - (n_d + 1)X/n_d]x_i$. The n_u upstream firms also compete in quantity and it is easily established that the only equilibrium is for each to produce $x_i = n_d/(n_d n_u + n_d + n_u + 1)$. Hence $X = n_u n_d/(n_d n_u + n_d + n_u + 1) = Y$, $w = (n_d + 1)/(n_d n_u + n_d + n_u + 1)$ and $p = (n_d + n_u + 1)/(n_d n_u + n_d + n_u + 1)$.

Allow now for the possibility of some upstream and downstream pairs of firms being vertically integrated. Let $m \leq \min\{n_u, n_d\}$ designate the number of such vertically integrated entities. At one extreme we may have $m = 0$. This is the situation just discussed, where there are no vertically integrated firms. At the other extreme is the situation where the maximum number of possible pairwise vertical integrations occur. We will call this full integration, which arises whenever $m = \min\{n_u, n_d\}$.

Consider a situation of full integration. We may then have either $n_d \leq n_u$ or $n_u < n_d$ and must discuss the two seperately. Whenever $n_d \leq n_u$, there will then be no demand for inputs from independent upstream firms and we simply have an n_d-firm Cournot oligopoly, with zero marginal costs. The equilibrium is symmetric and has each firm producing $y_i = 1/(n_d + 1)$. Hence $p = 1/(n_d + 1)$. If when m firms integrate we denote by $\tilde{\pi}(m, n_d, n_u)$ the equilibrium joint profits of an upstream and a downstream non integrated firms and by $\hat{\pi}(m, n_d, n_u)$ the total equilibrium profits of an integrated firm, then, from the above outcomes,

$$\tilde{\pi}(0, n_d, n_u) = \frac{n_d + n_d^2 + n_u^2}{(n_d + 1)^2 (n_u + 1)^2} \tag{3.1}$$

and

$$\hat{\pi}(n_d, n_d, n_u) = 1/(n_d + 1)^2. \tag{3.2}$$

Whenever $n_u < n_d$, we have to allow for the possibility that under full integration ($m = n_u$), the integrated firms may wish to continue supplying the independent downstream firms. This is best analysed as a limiting case of the situation where $m \in [1, n_u]$ firms are integrated, with no constraint on net purchases of the integrated firms from the non integrated sector.

Consider then the production equilibria when m firms are integrated, with the number of integrated firms satisfying $m \in [1, \min\{n_u, n_d - 1\}]$, so that at least one downstream firm is non integrated. It will be convenient to let the firms indexed i (without loss of generality, $i = 1, 2, \ldots, m$) be the ones that are integrated and let the firms indexed j be the ones that are not ($j = m+1, m+2, \ldots, n_u$ upstream and $j = m+1, m+2, \ldots, n_d$ downstream). Thus the integrated firms and the non integrated downstream firms simultaneously determine the quantities (y_i and y_j) of the final product in a final good production stage. This stage is preceded by the upstream production stage, during which the non integrated upstream firms and the integrated firms again compete in quantities taking into account the derived demand resulting from the final good production decisions of the next stage. The decision variable of the non integrated upstream firms is the quantity it produces of the upstream good, x_j. The decision that matters for the integrated firm in this stage is its net sales to the non integrated sector, which we will denote s_i. We will let the quantity of the upstream product traded between the non integrated firms and the integrated firms be determined endogenously with no a priori restrictions put on the direction of this trade. Thus individual integrated firms may, if they wish, choose to sell inputs to non integrated downstream firms or buy inputs from non integrated upstream firms and s_i may be either negative or positive. The total profit of an integrated firm is $(1-Y)y_i + ws_i$, the profit of a non integrated downstream firm is $(1-Y-w)y_j$ and that of a non integrated upstream firm is wx_j.

Taking into account the fact that at equilibrium the y_i's will be the same for all integrated firms and the y_j's will be the same for all non integrated downstream firms, the equilibrium conditions for the downstream production stage can be written

$$1-(m+1)y_i-(n_d-m)y_j=0 \tag{3.3}$$

$$1-my_i-(n_d-m+1)y_j=w, \tag{3.4}$$

from which we derive $y_i = (1+(n_d-m)w)/(n_d+1)$ and $y_j = (1-(m+1)w)/(n_d+1)$. Given that $Y = my_i + (n_d-m)y_i$, the market price of the final product will be $p = (1 + (n_d-m)w)/(n_d+1)$.

The market demand for the upstream product comes from the $n_d - m$ non integrated downstream firms. They will be supplied by the $n_u - m$ non integrated upstream firms, that produce x_j, $(j = m+1, \ldots, n_u)$, and potentially by the m integrated firms, that have *net* sales of input of s_i $(i = 1, \ldots, m)$. The competition at the upstream stage is therefore subject to the derived inverse demand[4]

$$w = \frac{1}{m+1}\left[1-\frac{n_d+1}{n_d-m}\left(\sum_{i=1}^{m} s_i + \sum_{j=m+1}^{n_u} x_j\right)\right]. \tag{3.5}$$

Since, again, the s_i's will be the same for each integrated firm, as will the x_j's for each non integrated upstream firm, the conditions that must be satisfied by the equilibrium of the

[4]Notice that the derived inverse demand is defined only for $m < n_d$.

upstream stage can be written

$$m-1-\left[\frac{(n_d+1)(m+1)^2}{n_d-m}-2m\right]s_i+\left[2(n_u-m)-\frac{(n_u-m)(n_d+1)(m+1)}{n_d-m}\right]x_j=0 \quad (3.6)$$

$$1-\frac{(n_d+1)(n_u-m+1)}{n_d-m}x_j-\frac{(n_d+1)m}{n_d-m}s_i=0. \quad (3.7)$$

These are derived respectively from the first-order condition of the typical integrated firm and that of the typical non integrated upstream firm. We discuss the solution in the Appendix.

Let $(s_i(m,n_d,n_u), x_j(m,n_d,n_u))$ denote the unique solution to (6) and (7). Substituting into (5), we get

$$w(m,n_d,n_u)=\frac{1}{m+1}\left[1-\frac{n_d+1}{n_d-m}\left[ms_i(m,n_d,n_u)+(n_u-m)x_j(m,n_d,n_u)\right]\right]. \quad (3.8)$$

The profit of an integrated firm will be

$$\hat{\pi}(m,n_d,n_u)=\left(\frac{1+(n_d-m)w(m,n_d,n_u)}{n_d+1}\right)^2+w(m,n_d,n_u)s_i(m,n_d,n_u) \quad (3.9)$$

while the joint profit of an upstream and downstream non integrated pair will be

$$\tilde{\pi}(m,n_d,n_u)=\left(\frac{1-(m+1)w(m,n_d,n_u)}{n_d+1}\right)^2+w(m,n_d,n_u)x_j(m,n_d,n_u) \quad (3.10)$$

The subgame perfect equilibrium profit of the production stages for $m\in[1,\min\{n_u,n_d-1\}]$ can be calculated by simply substituting for $w(m,n_d,n_u)$, $s_i(m,n_d,n_u)$ and $x_j(m,n_d,n_u)$. This includes $\hat{\pi}(n_u,n_d,n_u)$, the full integration profit when $n_u<n_d$,

4. The equilibria of the integration game

Consider now the integration decisions. These are assumed to be based on the profits expected from the subgame perfect equilibria of the upstream and downstream production stages, as just calculated. To simplify matters and avoid unnecessary repetition, we will think of the integration game as being played by the downstream firms when $n_d\le n_u$ and by the upstream firms when $n_u<n_d$. In the first case, each downstream firm decides whether to

integrate one upstream firm whereas in the second case each upstream firm decides whether to integrate one downstream firm.

It is useful to consider first the case of successive equal size oligopolies (i.e., $n_u = n_d = n$). In addition to serving as an important reference case, this will help in providing some insight into the nature of the equilibria that may arise. It is also useful in relating our results to some of the literature on vertical integration, since the often studied successive duopolies scenario is a special subcase. We will afterwards consider the case of $n_u \neq n_d$ and show how the relative size of the upstream and downstream industries matters.

4.1. The case of $n_u = n_d = n$

When $n_u = n_d = n$, then from (1) the consolidated profits of an upstream and a downstream pair of firms when no firm is integrated is

$$\tilde{\pi}(0, n, n) = \frac{(2n+1)n}{(n+1)^4} \tag{4.1}$$

and from (2), the profit of an integrated firm when all the firms are integrated is

$$\hat{\pi}(n, n, n) = 1/(n+1)^2. \tag{4.2}$$

Since $(2n+1)n/(n+1)^2 > 1$ for any $n \geq 2$, we immediately have $\tilde{\pi}(0, n, n) > \hat{\pi}(n, n, n)$ and hence

Proposition 1. *With successive n-firm oligopolies, the firms are always better off if none integrates than if they all integrate.*

In other words, in the situation where no one vertically integrates, the consolidated profits of a pair of upstream and downstream firms is always greater than the profit of the integrated pair when everyone integrates. In this sense, therefore, vertical integration never increases profits, no matter what the number of firms is.

In order to address the question as to which, if any, of those two situations can constitute a Nash-equilibrium of the integration game, we must make use of the production equilibria under asymmetric situations, where some but not all firms are integrated ($m \in [1, n-1]$). First note that, as shown in the Appendix from the solution for $s_i(m, n, n)$,

$$s_i \gtreqless 0 \quad \Leftrightarrow \quad \frac{m}{n-m} \gtreqless \frac{1}{2}\left[1 + \sqrt{1 + \frac{4(3(n-m)+1)}{(n-m)^2}}\right]. \tag{4.3}$$

Thus, somewhat surprisingly, s_i may be either negative or positive in this production equilibrium. If s_i is negative, then each integrated firm chooses to make *net purchases* of the input from the non integrated upstream firms at a positive price w even though it can supply itself internally at zero cost. The reason is that in doing so it pushes up the price of the upstream input for the non integrated downstream firms, thereby reducing competition at the downstream stage. The increase in equilibrium downstream profit this generates more than compensates the extra cost of the external supply. This is clearly a case of a "raising rivals' costs" strategy (Salop and Sheffman, 1983). As is clear from (13), this requires that the number of integrated firms be sufficiently small relative to the number of non integrated firms. A sufficient (though not necessary) condition for the cost raising strategy to work is that the number of non integrated firms be at least as large as the number of integrated firms[5].

When the number of non integrated firms is small relative to the number of integrated firms[6] then it does not pay to adopt a raising rivals' costs strategy and the integrated firms will choose to make positive net sales to the downstream non integrated firms. The reason is that when the non integrated downstream firms do not constitute a sufficiently important part of the downstream market, the relatively small gain from the reduction in competition to be had from raising their costs is spread across a relatively large number of integrated firms. The reduction in competition does not generate enough gains to the individual integrated firm to compensate its expense in supporting the cost raising strategy.

[5]The phenomenon of overbuying in the external input market after vertical integration in order to raise the rival's input costs has also been noted by Ayers (1987) in the context of successive duopolies.

[6]To take a few examples, with 1 out of $n \geq 4$ non integrated firms, 2 out of $n \geq 6$, 3 out of $n \geq 8$ or 4 out of $n \geq 11$, then $s_j > 0$.

Using (9) and (10) and the solutions for $w(n,n,n)$, $s_i(n,n,n)$ and $x_j(n,n,n)$ we can state a number of propositions characterizing the integration decisions in the case where $n_d = n_u = n$. The tedious algebra involved in their proofs is relegated to the Appendix.

Proposition 2. *When $n_u = n_d = n$, there always exists an equilibrium where all firms are vertically integrated.*

Proof: The proof involves showing that for all $n \geq 2$, $\hat{\pi}(n,n,n) \geq \tilde{\pi}(n-1,n,n)$. The equality in fact holds strictly, so that vertical integration always increases the joint profits of a pair of firms if all the other pairs of firms are already integrated. ■

Proposition 3. *When $n_u = n_d = n$, then, if and only if $n \geq 5$, there exists an equilibrium where no firms are vertically integrated.*

Proof: The proof involves showing that $\tilde{\pi}(0,n,n) \geq \hat{\pi}(1,n,n)$ for all $n \geq 5$. Again, the inequality turns out to hold strictly. ■

Proposition 4. *When $n_u = n_d = n$, to vertically integrate is a dominant strategy when $n \leq 4$, but not when $n \geq 5$.*

Proof: That vertical integration is not a dominant strategy for $n \geq 5$ follows as a corollary of Propositions 3 and 1. That it is for $n \leq 4$ involves showing that $\hat{\pi}(m+1,n,n) > \tilde{\pi}(m,n,n)$, for all $m \in [0, n-1]$ and $n = 2,3,4$. ■

Proposition 4 means that if $n_u = n_d = n \leq 4$, it is always profitable for an upstream-downstream pair of firms to integrate no matter what the other firms do. It follows that for all firms to integrate is the unique equilibrium in that situation. By Proposition 1, firms then face a prisoner's dilemma: although full integration is the unique equilibrium, each would be better off if no one would integrate. However, as the number of firms increases, the downstream mark-up falls and so does the gain from *unilaterally* integrating a pair of firms. This explains why, as stated in Proposition 3, it also is an equilibrium for no one to integrate if $n_u = n_d \geq 5$. By proposition 1, what was a prisoner's dilemma for $n \leq 4$ therefore becomes a coordination problem for $n \geq 5$: although everyone is better off if no one integrates, this is not the unique equilibrium.

For $n \geq 5$, we have just shown that there are at least two equilibria, both symmetric ($m = 0$ and $m = n$). There remains the possibility of some equilibria with $m \in [1, n-1]$ when $n \geq 5$. An equilibrium where m pairs of firms are integrated and $n - m$ pairs are not, with $m \in [1, n-1]$, will exist if and only if

$$\hat{\pi}(m, n, n) \geq \tilde{\pi}(m-1, n, n) \quad \text{and} \quad \hat{\pi}(m+1, n, n) \leq \tilde{\pi}(m, n, n).$$

Define $D(z, n) = \hat{\pi}(z+1, n, n) - \tilde{\pi}(z, n, n)$. It follows from the above two conditions that for such an equilibrium to exist, there must be some admissible z^* such that $D(z^* - 1, n) \geq 0$ and $D(z^*, n) \leq 0$. It can be verified numerically that, for $n \in [5, 1000]$, this is not the case. In fact, for n in that range, $D(z, n) - D(z-1, n) > 0$ for all admissible z. It seems safe to conjecture that when $n_u = n_d = n$, there exist no asymmetric equilibria, where some but not all firms vertically integrate.

All of the above propositions have been obtained in a context where the net sales to the non integrated firms by the integrated firms is determined endogenously. Since complete foreclosure ($s_i = 0$) is often simply assumed in the analysis of successive oligopolists[7], it seems appropriate to briefly consider what is the impact of such an assumption[8]. This can be summarized in the following proposition.

Proposition 5. *When $n_u = n_d = n$, if complete foreclosure is exogenously imposed then there always exists an equilibrium where all the firms are integrated and there never exists an equilibrium where none of the firms are integrated.*

[7]Amongst the papers cited here, for instance, this is true of Bonanno and Vickers (1988), Greenhut and Ohta (1979), Lin (1988), Salinger (1988), and Nakache and Soubeyran (1989a and 1989b).

[8]It can be tempting to assume at least $s_i \geq 0$, as is implicitly the case in Ordover, Saloner and Salop (1990) and Hart and Tirole (1990). At first thought this assumption appears innocuous, since the integrated firm can supply itself internally at zero marginal cost and hence raises its own cost by choosing $s_i < 0$. But, as we have shown above, this neglects any gains which may arise from the effect on the downstream rivals' competitiveness of setting $s_i < 0$.

Proof: The details of the proof are left for the Appendix. ■

Numerical simulations (for $n \in [2, 1000]$) also lead us to conjecture that to integrate vertically always is a dominant strategy when foreclosure is imposed, so that the equilibrium where all n pairs of firms integrate would be the only equilibrium to the integration game. Proposition 1 obviously still holds, since, with $n_d = n_u = n$, foreclosure changes neither $\hat{\pi}(m, n, n))$ nor $\tilde{\pi}(0, n, n)$.

To understand why the situation where no firm integrates would not be an equilibrium any more, recall that if a single pair of firms chooses to integrate when foreclosure is not imposed, then it would choose to make net purchases from the non integrated upstream firms, i.e., set s_i negative. This follows from (13). But this means that if this lone integrated firm could *credibly* commit to $s_i = 0$, it would move itself closer to the outcome it would choose were it to be a Stackelberg leader in the upstream stage, which can be verified to involve $s_i > 0$ in that case. It would therefore make greater profits. This is exactly what the exogenous constraint accomplishes, thereby sufficiently raising the profits that a lone integrated firm can realize to make it attractive to unilaterally deviate when it otherwise was not, i.e., when n was five or more. Hence whether or not complete foreclosure is assumed at the outset can have important consequences for the type of equilibria that may arise in the integration game. This is true even when, as is the case with $n_u = n_d = n$, the eventual outcome of this game does not involve coexistence of integrated and non integrated firms. But just as the Stackelberg outcome requires that the firm be able to credibly precommit to it, so would, in a situation which dictates $s_i < 0$ in equilibrium, a strategy of foreclosure.

Finally, a word is in order about the perhaps over-studied successive duopolies case. In that case, the unique equilibrium is for the two upstream-downstream pairs of firms to vertically integrate. It in fact always pays for one pair of firms to integrate no matter what the other pair does. Each would however be better off if no one would integrate and thus the firms face a prisoner's dilemma. Finally, the impact of modelling the vertical integration problem with $s_i = 0$ is then not apparent: although the equilibrium outputs will be changed by this assumption, full integration remains a unique equilibrium. Therefore to assume successive duopolies certainly generalizes in important respects the succesive monopolies case, since it permits strategic considerations which are otherwise absent. However, from the above propositions, it is clear that setting $n_u = n_d = 2$ involves some loss of generality.

4.2. The case of $n_u \neq n_d$

We now allow for different numbers of firms downstream and upstream. Two subcases are relevant: that in which $n_u > n_d$ and that in which $n_d > n_u$. The general solution for production and sales decisions and for input price is provided in the Appendix. The thrust of the effect of having $n_u \neq n_d$ can however be observed by considering the case where $\min\{n_u, n_d\} = 2$. We will mainly restrict ourselves here to that case.

Consider first the case where $n_d > n_u = 2$. The two upstream firms simultaneously decide whether to integrate one downstream firm on the basis of the anticipated subgame perfect equilibrium profits of the two production stages. The main results of the integration game in that case can be summarized in the following proposition.

Proposition 6. *When $n_d > n_u = 2$, then*

1. *The profit of an integrated firm under full integration ($m = 2$) is always less than the consolidated profit of a non integrated pair of firms when no one integrates ($m = 0$).*

2. *For $n_d = 3$, to integrate is a dominant strategy and full integration ($m = 2$) is the only equilibrium.*

3. *For $n_d = 4$, the only equilibria are for one upstream firm to integrate ($m = 1$), with the other one remaining non integrated.*

4. *For $n_d \geq 5$, the only equilibrium is where no firm integrates ($m = 0$).*

Proof: The full payoff matrices of the integration game are calculated in the Appendix for $n_d = 3, 4, 5$ and can be seen to confirm 6.2, 6.3 and 6.4. It is also shown there that $\tilde{\pi}(0, n_d, 2) > \hat{\pi}(2, n_d, 2)$ for all $n_d \geq 2$ — which proves 6.1 — and that $\tilde{\pi}(0, n_d, 2) > \hat{\pi}(1, n_d, 2)$ for all $n_d \geq 5$ — which completes the proof of 6.4. ■

The possibilities are therefore richer than in the case where $n_d = n_u = n \geq 2$. It is still true that everyone is better off when no one vertically integrates. But now this may be the only equilibrium. Indeed when n_d is sufficiently large, it does not pay to integrate whenever everyone else is integrated, as was always the case with $n_d = n_u = n \geq 2$. Thus there are now situations where, from all perspectives, vertical integration reduces joint profits.

A somewhat striking result is that even though both upstream and downstream firms are perfectly identical in all respects, asymmetric equilibria, where some but not all upstream firms vertically integrate, may now arise in the integration game. The situation with $n_u = 2$ and $n_d = 4$ (see Proposition 6.3) is a case in point: in equilibrium, either upstream firm 1 is integrated and upstream firm 2 is not, or vice-versa. Thus differences in the decision to vertically integrate can arise in equilibrium even though there are no inherent differences among firms[9].

It is important to note that, with $n_u = 2$, the net sales of input by an integrated firm are now given by:

$$s_i = \begin{cases} \dfrac{4(1 - n_d)}{(1 + n_d)(8 + 4n_d)} & \text{if } m = 1 \\ \dfrac{n_d - 2}{17 + 5n_d} & \text{if } m = 2 \end{cases} \tag{4.4}$$

This means that with $n_d \geq 3$, when, as predicted by Proposition 6.2, all the upstream firms integrate ($m = 2$), they will choose to continue supplying the independent downstream firms ($s_i > 0$). The profits from the sales of the input at a price which exceeds marginal cost more than offsets the reduction in profits resulting from the increased downstream competition.

[9]This consequence of the strategic effect is not unlike that found recently in a vertically related framework by Besanko and Perry (1993), when studying the equilibrium incentives by manufacturers to adopt exclusive dealing. They find that differences in the extent to which firms adopt exclusive dealing can arise in an equilibrium among identical manufacturers.

This also means that when only one of the upstream firms is integrated, as will be the case in equilibrium with $n_d = 4$ (Proposition 6.3), it will always choose to raise its downstream rivals' costs by purchasing inputs from the remaining non integrated upstream firm ($s_i < 0$). The case of $n_u = 2$ and $n_d = 4$ therefore provides an example where the raising rivals' cost strategy of setting $s < 0$ would actually be observed as an outcome of an asymmetric integration equilibrium. This is in contrast with situations where $n_d = n_u = n$, as in the case, for instance, of successive duopolies. The only subgame perfect equilibria to the integration game were then symmetric and therefore $s_i < 0$ could only occur as an outcome of an (out of perfect equilibrium) production subgame.

The situation is much more favorable to integration in the case where $n_u > n_d$ than in the case where $n_d > n_u$. If full integration then occurs ($m = n_d$), the market structure is considerably altered, since the independent upstream firms are left with no demand for their product and have no choice but to shut down, leaving the integrated firms in control of both stages of production. As a result we have the following:

Proposition 7. *When $n_u > n_d = 2$, then*

1. *For all $n_u \geq 3$, the profit of an integrated firm under full integration ($m = 2$) is always greater than the consolidated profit of a non integrated pair of firms when no one integrates ($m = 0$).*

2. *Full integration ($m = 2$) is the only equilibrium. It is in fact an equilibrium in dominant strategies.*

Proof: It is shown in the Appendix that $\hat{\pi}(2, 2, n_u) > (<)\ \tilde{\pi}(0, 2, n_u)$ for $n_u \geq 3$ ($n_u = 2$) — which proves 7.1 — and that $\hat{\pi}(2, 2, n_u) > \tilde{\pi}(1, 2, n_u)$ and $\hat{\pi}(1, 2, n_u) > \tilde{\pi}(0, 2, n_u)$ for all $n_u \geq 2$ — which proves 7.2. ∎

Part 1 of the Proposition is somewhat misleading since it suggests that the prisoner's dilemma can occur only when $n_u \leq n_d$. This is not the case. For $n_d > 2$, there is a range a values of $n_u > n_d$ such that $\hat{\pi}(n_d, n_d, n_u) < \tilde{\pi}(0, n_d, n_u)$. But as n_u gets large enough[10], it will be the case that $\hat{\pi}(n_d, n_d, n_u) > \tilde{\pi}(0, n_d, n_u)$. Beyond this threshold value of n_u (which increases with n_d), vertical integration increases the joint profit, from whichever perspective we look at it: to integrate is a dominant strategy and each integrated pair is better off if all integrate.

If only one firm were integrated, it would choose to purchase some of its inputs from the non integrated upstream suppliers ($s_i = 2(1 - n_u)/3(2 + 3n_u) < 0$). But as already noted, under full integration ($m = n_d$), $s_i = 0$ in this case. This means that if one had assumed complete foreclosure ($s_i \equiv 0$), part 1 of Proposition 7 would still hold. Furthermore, as argued in section 4.1, if only one firm were integrated, complete foreclosure would increase the integrated firm's profit relative to the simultaneous equilibrium outcome of the upstream stage, by moving it towards the Stackelberg outcome. It follows that if to integrate was a dominant strategy with $s_i < 0$ it must also be a dominant strategy under complete foreclosure and part 2 of Proposition 7 holds. To have imposed complete foreclosure when modelling the production game would therefore have been of no consequence in this case for the types of equilibria that may arise from the integration game.

Things are quite different when $n_d > n_u$, as it was with $n_d = n_u$. In that case, to assume complete foreclosure can have drastic effects on the type of integration equilibria that may arise. To illustrate, assume $n_u = 2$ as in Proposition 6. Then, for $n_d \geq 3$, one verifies (see Appendix, part G) that part 1 of Proposition 6 does not hold[11]. One also verifies that no asymmetric equilibria ($m = 1$) occur. Furthermore, while for $n_d = 3, 4, 5$, full integration ($m = 2$) is the only equilibrium, both full integration and no integration ($m = 0$) can coexist as equilibria, as the case of $n_d = 6$ illustrates.

[10]For example, when $n_d = 3$, this occurs for $n_u \geq 6$. As one would expect, as n_u tends to infinity and the upstream mark-up when no one is integrated becomes negligible, $\hat{\pi}(n_d, n_d, n_u) - \tilde{\pi}(0, n_d, n_u)$ tends to zero, for there can then be no gain from integrating.

[11]It can hold for $n_u > 2$ if n_d is sufficiently large.

5. Conclusion

Our results show that, generally, whether joint profits increase or not when an upstream and a downstream firm merge in the presence of double marginalization does depend on how we formulate the question. Our results also show that it can depend crucially on the number of upstream and downstream firms there are in the industry initially and on the way we model the market interaction of integrated and non integrated firms. We have shown that when it is assumed, as we do, that integrated firms may freely trade with downstream and upstream non integrated firms, they may surprisingly choose to purchase inputs from independent upstream firms at a price which exceeds the marginal production cost of their own upstream division. This raising rivals' costs strategy is a phenomenon which has some impact on the nature of the integration equilibria, whether or not these equilibria involve coexistence of integrated and non integrated firms. We show that such asymmetric equilibria may in fact arise even when firms are identical.

Appendix

A. The solutions for $s_i(m, n_d, n_u)$ and $x_j(m_d, n_u)$, $1 \le m \le \min\{n_u, n_d - 1\}$

Let

$$\begin{aligned}
A &= \frac{(n_d+1)(m+1)^2}{n_d - m} - 2m > 0 \\
B &= 2(n_u - m) - \frac{(n_u - m)(n_d+1)(m+1)}{n_d - m} < 0 \\
C &= m - 1 \ge 0 \\
D &= \frac{n_d - m}{m(n+1)} > 0 \\
E &= \frac{n_u - m + 1}{m} > 0.
\end{aligned}$$

Then the first-order conditions (6) and (7) are written

$$C - As_i + Bx_j = 0 \tag{A–1}$$

$$D - s_i - Ex_j = 0 \tag{A–2}$$

and the unique solution is

$$(x_j(m, n_d, n_u), s_i(m, n_d, n_u)) = \left(\frac{AD - C}{AE + B}, \frac{CE + BD}{AE + B}\right). \tag{A–3}$$

We first verify that the denominator is positive:

$$AE + B = [((n_u - m)(m+1) + 1)(n_d + 1) + ((n_d + 3)m + 2)m]/m(n_d - m) > 0.$$

This assures the stability of the solution, in the sense that the slope of the $x_j(s_i)$ curve (from (A–2)) drawn in (x_j, s_i)-space is steeper than that of the $s_i(x_j)$ curve (from (A–1)). Furthermore,

$$AD - C = [(m+1)(n_d + 2m + 1)]/m(n_d + 1) > 0$$

which assures that $x_j(m, n_d, n_u)$ is positive.

As for the sign of $s_i(m, n_d, n_u)$, it will be the same as that of $CE + BD$. When $n_d = n_u = n$,

$$\begin{aligned}
CE + BD &= \frac{m^2 - (n-m)m - 3(n-m) - 1}{m(n+1)} \\
&= \frac{k^2 - k - [3(n-m)+1]/(n-m)^2}{k(n+1)} = \frac{(k - \lambda_1)(k - \lambda_2)}{k(n+1)}
\end{aligned}$$

where $k = m/(n-m)$ and

$$\lambda_1 = \frac{1}{2}\left[1 + \sqrt{1 + \frac{4(3(n-m)+1)}{(n-m)^2}}\right] > 0, \qquad \lambda_2 = \frac{1}{2}\left[1 - \sqrt{1 + \frac{4(3(n-m)+1)}{(n-m)^2}}\right] < 0.$$

Since $k > 0$, $k - \lambda_2 > 0$. Therefore $\text{sign}(s_i) = \text{sign}(k - \lambda_1)$, or

$$s_i(m,n,n) \gtreqless 0 \quad \Leftrightarrow \quad \frac{m}{n-m} \gtreqless \frac{1}{2}\left[1 + \sqrt{1 + \frac{4(3(n-m)+1)}{(n-m)^2}}\right].$$

More generally,

$$s_i(m, n_d, n_u) = (m - n_d)(1 - 3m - 2m^2 - (m-1)n_d + 2(m+1)n_u)/\Delta \tag{A–4}$$

and

$$x_j(m, n_d, n_u) = (n_d - m)(m+1)(n_d + 2m + 1)/\Delta \tag{A–5}$$

where

$$\Delta = (n_d + 1)(1 + m + 2m^2 - (m-1)n_d + (m+1)(n_d+1)n_u) > 0.$$

Substituting (A–4) and (A–5) into (8), we get

$$w(m, n_d, n_u) = (n_d + 1)(n_d + 2m + 1)/\Delta. \tag{A–6}$$

B. Proof of Proposition 2

We know that $\hat{\pi}(n,n,n) = 1/(n+1)^2$. From (10),

$$\begin{aligned}
\tilde{\pi}(n-1,n,n) &= \left(\frac{1 - nw(n-1,n,n)}{n+1}\right)^2 + w(n-1,n,n)x_j(n-1,n,n) \\
&= \hat{\pi}(n,n,n) + \frac{w(n-1,n,n)}{(n+1)^2}\left[n^2 w(n-1,n,n) - 2n + (n+1)^2 x_j(n-1,n,n)\right].
\end{aligned}$$

Therefore $\hat{\pi}(n,n,n) \geq \tilde{\pi}(n-1,n,n)$ if and only if

$$n(nw(n-1,n,n) - 2) + (n+1)^2 x_j(n-1,n,n) \leq 0, \tag{A–1}$$

where $s_j(n-1,n,n)$, $x_j(n-1,n,n)$ and $w(n-1,n,n)$ can be obtained directly from (A–4), (A–5) and (A–6). After substitution and some simplifications, we find that condition (A–7) reduces easily to $n[n(2n^2 - 2n - 3) + 5] \geq 0$, which holds strictly for all $n \geq 2$. Q.E.D.

C. Proof of Proposition 3

We know that $\tilde{\pi}(0,n,n) = (2n+1)n/(n+1)^4$. After substituting for the values of $s_i(1,n,n)$ and $w(1,n,n)$ obtained from (A–4) and (A–5) into (9), $\hat{\pi}(1,n,n) = (5n^3 + 11n^2 + 27n - 11)/4(n+1)(n^2+n+2)^2$ and

$$\tilde{\pi}(0,n,n) - \hat{\pi}(1,n,n) = \frac{[3n^6 + 22n + 11] - [6n^5 + 27n^4 + 56n^3 + 11n^2]}{4(n+1)(n^2+n+2)^2}. \tag{A–1}$$

Both terms in brackets in the numerator are positive. One easily verifies numerically that the first term is smaller than the second one for $n = 2, 3, 4$ but that it is larger for $n = 5$. Since n^6 is the term with the largest exponent, this must also be the case for all $n > 5$. Hence (A–8) is positive for all $n \geq 5$. Q.E.D.

D. Proof of Proposition 5

When complete foreclosure ($s_i = 0$) is imposed, the solution is exactly that obtained in Salinger(1988), which in our notation gives:

$$y_i(m, n_d, n_u) = \frac{1 + (n_d - m)/(m+1)(n_u - m + 1)}{n_d + 1} \tag{A–1}$$

$$y_j(m, n_d, n_u) = \frac{1 - 1/(n_u - m + 1)}{n_d + 1} \tag{A–2}$$

and

$$x_j(m, n_d, n_u) = \frac{n_d - m}{(n_d + 1)(n_u - m + 1)} \tag{A–3}$$

from which

$$w(m, n_d, n_u) = \frac{1}{(m+1)(n_u - m + 1)} \tag{A–4}$$

and

$$p(m, n_d, n_u) = \frac{1 + (n_d - m)/(m+1)(n_u - m + 1)}{n_d + 1}. \tag{A–5}$$

It is then straightforward to calculate $\hat{\pi}^0(m, n_d, n_u)$ and $\tilde{\pi}^0(m, n_d, n_u)$, by simple substitution. The superscript 0 indicates that the equilibrium profit is now calculated with foreclosure imposed. Because $s_i = 0$, whenever $m = \min\{n_u, n_d\}$ we now simply have an m-firm Cournot oligopoly with zero marginal cost and hence profit is $1/(m+1)^2$, as was the case only if $n_d \leq n_u$ with no foreclosure. When $m = 0$, the profit is of course the same as with no foreclosure.

Hence with $n_d = n_u = n$, $\hat{\pi}^0(n, n, n) = 1/(n+1)^2$ while

$$\tilde{\pi}^0(n-1, n, n) = \frac{1}{(n+1)^2}\left(\frac{2n+1}{4n}\right) = \hat{\pi}^0(n, n, n)\left(\frac{2n+1}{4n}\right) < \hat{\pi}^0(n, n, n)$$

and it does not pay to unilaterally deviate when all firms are integrated.

We also know that

$$\tilde{\pi}^0(0, n, n) = \frac{(2n+1)n}{(n+1)^4} = \hat{\pi}^0(n, n, n)\left(\frac{(2n+1)n}{(n+1)^2}\right)$$

while

$$\hat{\pi}^0(1, n, n) = \frac{1}{(n+1)^2}\left(\frac{3n-1}{2n}\right)^2 = \hat{\pi}^0(n, n, n)\left(\frac{3n-1}{2n}\right)^2$$

from which we get that $\hat{\pi}^0(1, n, n) > \tilde{\pi}^0(0, n, n)$ if and only if $n^4 + 8n^3 - 2n^2 - 4n + 1 > 0$, a condition which holds for all $n \geq 2$. Therefore it always pays to be the first one to unilaterally integrate. Q.E.D.

E. Proof of Proposition 6

By substituting (A–4), (A–5) and (A–6) into (9) and (10) and evaluating at $n_u = 2$, we find that $\tilde{\pi}(0, n_d, 2) = (n_d^2 + n_d + 4)/9(n_d + 1)^2$ and $\hat{\pi}(2, n_d, 2) = (2n_d^2 + 17n_d + 39)/(5n_d + 17)^2$. Subtracting the two, we get

$$\tilde{\pi}(0, n_d, 2) - \hat{\pi}(2, n_d, 2) = \frac{7n_d^4 + 6n_d^3 - 116n_d^2 + 114n_d + 805}{9(n_d + 1)^2(5n_d + 17)^2}$$

which is positive for all $n_d \geq 2$, thus proving part 1 of the Proposition.

The payoff matrices for the integration game, again normalized by setting $\hat{\pi}(2, 2, 2) = 1000$ and rounded to the nearest integer, are, for $n_d = 3, 4, 5$,

$n_d = 3$:

		Firms 2: Integrated	Firms 2: Not integrated
Firms 1	Integrated	949, 949	1170, 900
	Not integrated	900, 1170	1000, 1000

$n_d = 4$:

		Firms 2: Integrated	Firms 2: Not integrated
Firms 1	Integrated	914, 914	1003, 981
	Not integrated	981, 1003	960, 960

$n_d = 5$:

		Firms 2: Integrated	Firms 2: Not integrated
Firms 1	Integrated	888, 888	903, 1026
	Not integrated	1026, 903	944, 944

This proves part 2 and 3. To complete the proof of part 4, we verify that $\hat{\pi}(1, n_d, 2) = (n_d^3 + 7n_d^2 + 27n_d + 37)/16(n_d + 1)(n_d + 2)^2$ and

$$\tilde{\pi}(0, n_d, 2) - \hat{\pi}(1, n_d, 2) = \frac{7n_d^4 + 8n_d^3 - 114n_d^2 - 256n_d - 77}{144(n_d + 1)^2(n_d + 2)^2}$$

which is positive for $n_d = 2, 3, 4$ but negative for $n_d \geq 5$, at which point it becomes unprofitable to unilaterally deviate when no one is vertically integrated. Q.E.D.

F. Proof of Proposition 7

Again, by substituting (A–4), (A–5) and (A–6) into (9) and (10) and evaluating now at $n_d = 2$, we find that $\hat{\pi}(2, 2, n_u) = 1/9$ while $\tilde{\pi}(0, 2, n_u) = (n_u^2 + 6)/9(n_u + 1)^2$, so that

$$\tilde{\pi}(0, n_d, 2) - \hat{\pi}(2, n_d, 2) = \frac{2n_u - 5}{9(n_u + 1)^2}$$

which is obviously negative for $n_u = 2$ but positive for $n_u \geq 3$, thus proving part 1 of the Proposition.

Similarly, $\tilde{\pi}(1, 2, n_u) = (16n_u^2 - 12n_u + 31)/6(3n_u + 2)^2$ and $\hat{\pi}(1, 2, n_u) = (12n_u^2 + 16n_u + 47)/12(3n_u + 2)^2$, which means that

$$\hat{\pi}(2, 2, n_u) - \tilde{\pi}(1, 2, n_u) = \frac{5(12n_u - 17)}{18(3n - u + 2)^2} > 0 \quad \text{for all} \quad n_u \geq 2$$

and

$$\hat{\pi}(1, 2, n_u) - \tilde{\pi}(0, 2, n_u) = \frac{72n_u^3 + 41n_u^2 + 41n_u + 45}{36(n_u + 1)^2(3n_u + 2)^2} > 0 \quad \text{for all} \quad n_u > 0.$$

Therefore it always pays to vertically integrate no matter what the other does. Q.E.D.

G. Foreclosure when $n_d > n_u = 2$

The payoff matrices of the integration game with foreclosure imposed and $n_d > n_u = 2$ can be calculated by using (A–9) to (A–13). For $n_d = 3, 4, 5, 6$, they are:

$n_d = 3$:

		Firms 2: Integrated	Firms 2: Not integrated
Firms 1	Integrated	1000, 1000	1267, 703
	Not integrated	703, 1267	1000, 1000

$n_d = 4$:

		Firms 2: Integrated	Firms 2: Not integrated
Firms 1	Integrated	1000, 1000	1103, 765
	Not integrated	765, 1103	960, 960

$n_d = 5$:

		Firms 2: Integrated	Firms 2: Not integrated
Firms 1	Integrated	1000, 1000	1000, 813
	Not integrated	813, 1000	944, 944

$n_d = 6$:

		Firms 2: Integrated	Firms 2: Not integrated
Firms 1	Integrated	1000, 1000	930, 850
	Not integrated	850, 930	939, 939

References

Ayers, Ian (1987) "Essays on Vertical Foreclosure, Cartel Stability and the Structural Determinants of Oligopolistic Behavior," Unpublished Ph.D. Dissertation, Massachusetts Institute of Technology, Cambdrige, Massachusetts.

Besanko, David and Martin K. Perry (1993) "Equilibrium incentives for exclusive dealing in a differentiated products oligopoly," *RAND Journal of Economics*, **24**, 646–667.

Bonanno, Giacomo and John Vickers (1988) "Vertical Separation," *The Journal of Industrial Economics*, **36**, 257–265.

Greenhut, M. L. and H. Ohta (1979) "Vertical Integration of Successive Oligopolists," *American Economic Review*, **69**, 137–141.

Hart, Oliver and Jean Tirole (1990) "Vertical Integration and Market Foreclosure," *Brookings Papers on Economic Activity : Microeconomics*, , 205–286.

Krattenmaker, Thomas G. and Steven C. Salop (1986) "Anticompetitive Exclusion: Raising Rivals' Costs to Achieve Power over Price," *Yale Law Journal*, **96**, 209–293.

Lin, Y. Joseph (1988) "Oligopoly and Vertical Integration: Note," *American Economic Review*, **78**, 251–254.

Nakache, Yves and Antoine Soubeyran (1989a) "Vertical Integration, Product and Cost Differentiation: The Case of Bertrand," G.R.E.Q.E., Universit Aix-Marseille II.

Nakache, Yves and Antoine Soubeyran (1989b) "Intgration verticale, diffrenciation des produits et disparit des cots: le cas de Cournot," G.R.E.Q.E., Universit Aix-Marseille II.

Ordover, Janusz A., Saloner, Garth and Steven C. Salop (1990) "Equilibrium Vertical Foreclosure," *American Economic Review*, **80**, 127–142.

Perry, Martin K. (1989) "Vertical Integration: Determinants and Effects," in *Handbook of Industrial Organization*, ed. Richard Schmalensee and Robert D. Willig. New York: North Holland.

Salinger, Michael A. (1988) "Vertical Mergers and Market Foreclosure," *Quarterly Journal of Economics*, **103**, 345–356.

Salop, Steven C. and David Scheffman (1983) "Raising Rival's Costs," *American Economic Review*, **73**, 267–271.

Tirole, Jean (1989) *The Theory of Industrial Organization*. Cambridge: MIT Press.

Liste des publications au CIRANO

Cahiers CIRANO / *CIRANO Papers* (ISSN 1198-8169)

94c-1 Faire ou faire faire : La perspective de l'économie des organisations / par Michel Patry

94c-2 Commercial Bankruptcy and Financial Reorganization in Canada / par Jocelyn Martel

94c-3 L'importance relative des gouvernements : causes, conséquences, et organisations alternatives / par Claude Montmarquette

95c-1 La réglementation incitative / par Marcel Boyer

95c-2 Anomalies de marché et sélection des titres au Canada / par Richard Guay, Jean-François L'Her et Jean-Marc Suret

Série Scientifique / *Scientific Series* (ISSN 1198-8177)

95s-25 Matching, Human Capital, and the Covariance Structure of Earnings / par Daniel Parent

95s-26 Industry-Specific Capital and the Wage Profile: Evidence from the NLSY and the PSID / par Daniel Parent

95s-27 Wages and Mobility: The Impact of Employer-Provided Training / par Daniel Parent

95s-28 Survol des contributions théoriques et empiriques liées au capital humain / par Daniel Parent

95s-29 Heterogeneous Expectations, Short Sales Relation and the Risk-Return Relationship / par Jean-François L'Her et Jean-Marc Suret

95s-30 L'impact de la réglementation en matière de santé et sécurité du travail sur le risque d'accident au Québec : de nouveaux résultats / par Paul Lanoie et David Stréliski

95s-31 Stochastic Volatility and Time Deformation: An Application to Trading Volume and Leverage Effects / par Eric Ghysels et Joanna Jasiak

95s-32 Market Time and Asset Price Movements Theory and Estimation / par Eric Ghysels, Christian Gouriéroux et Joanna Jasiak

95s-33 Real Investment Decisions Under Information Constraints / par Gérard Gaudet, Pierre Lasserre et Ngo Van Long

95s-34 Signaling in Financial Reorganization: Theory and Evidence from Canada / parJocelyn Martel

95s-35 Capacity Commitment Versus Flexibility: The Technological Choice Nexus in a Strategic Context / Marcel Boyer et Michel Moreaux

95s-36 Some Results on the Markov Equilibria of a class of Homogeneous Differential Games / Ngo Van Long et Koji Shimomura

95s-37 Dynamic Incentive Contracts with Uncorrelated Private Information and History Dependent Outcomes / Gérard Gaudet, Pierre Lasserre et Ngo Van Long

95s-38 Costs and Benefits of Preventing Worplace Accidents: The Case of Participatory Ergonomics / Paul Lanoie et Sophie Tavenas

95s-39 On the Dynamic Specification of International Asset Pricing Models / Maral kichian, René Garcia et Eric Ghysels

95s-40 Vertical Integration, Foreclosure and Profits in the Presence of Double Marginalisation / Gérard Gaudet et Ngo Van Long