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René Garcia, Éric Renault

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A Note on Hedging in ARCH and Stochastic Volatility Option Pricing Models*

René Garcia†, *Éric Renault*‡

Résumé / Abstract

Duan (1995) a proposé récemment une formule de valorisation d'option fondée sur un modèle GARCH ainsi que la formule de couverture correspondante. Dans un modèle similaire de type ARCH pour l'actif sous-jacent conduisant à la même formule de valorisation, Kallsen et Taqqu (1994) arrivent à une formule de couverture différente. Dans cette note, nous expliquons cette différence en soulignant que la formule de Kallsen et Taqqu correspond au concept usuel de couverture dans le cadre des modèles de type ARCH. Nous trouvons toutefois que la formule de couverture de Duan a un certain attrait et proposons un modèle de volatilité stochastique qui en assure la validité. Nous concluons par une comparaison des modèles ARCH et de volatilité stochastique pour la valorisation d'options.

Recently, Duan (1995) proposed a GARCH option pricing formula and a corresponding hedging formula. In a similar ARCH-type model for the underlying asset, Kallsen and Taqqu (1994) arrive at a hedging formula different from Duan's , although they concur on the pricing formula. In this note, we explain the difference by pointing out that the formula developped by Kallsen and Taqqu corresponds to the usual concept of hedging in the context of ARCH-type models. We argue however that Duan's formula has some appeal and propose a stochastic volatility model which ensures its validity. We conclude by a comparison of ARCH-type and stochastic volatility option pricing models.

Mots Clés : Valorisation d'options avec modèle GARCH, propriété d'homogénéité, volatilité implicite de Black-Scholes

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* Correspondence Address: René Garcia, CIRANO, 2020 University Street, 25th floor, Montréal, Qc, Canada H3A 2A5 Tel: (514) 985-4014 Fax: (514) 985-4039 e-mail: garciar@cirano.umontreal.ca
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† Université de Montréal, C.R.D.E. and CIRANO

‡ Université des Sciences Sociales de Toulouse, GREMAQ and IDEI, Institut Universitaire de France

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Introduction

Recently, Duan (1995) and Kallsen and Taqqu (1994) (hereafter referred to as KT) proposed option pricing and hedging formulas when the underlying asset follows a ARCH-type process. Duan derives his formula in a discrete-time equilibrium model, while KT develop theirs in a no-arbitrage continuous time setting. Although their different approaches lead to the same pricing formula, they do not concur on the hedging formula. In this note, we explain this difference by pointing out that the formula developed by KT corresponds to the usual concept of hedging in the context of ARCH-type models. We argue however that Duan's simpler formula has some appeal for practical implementation and propose a stochastic volatility model which ensures its validity while preserving the nice features of the GARCH option pricing model, most notably the replication of the implicit volatility[1] smile effect. In conclusion, we stress that ARCH-type and stochastic volatility option pricing models should not be seen as competitors (as it is commonly believed) but rather as complements, since the ARCH model offers a useful discrete-time filter for SV models.

1 Option Pricing and Hedging in ARCH Models

To point out the difference between Duan and KT hedging formulas, we briefly summarize their respective approaches to option pricing when the underlying asset follows a GARCH process.

Duan considers a discrete-time economy in which the one-period rate of return $\ln \frac{X_t}{X_{t-1}}$ is assumed to be conditionally (given the information Φ_{t-1}) normally distributed under probability measure P:

$$\ln \frac{X_t}{X_{t-1}} = r + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \epsilon_t \tag{1}$$

where ϵ_t has mean zero and conditional variance h_t under P; r and λ denote as usual the constant risk-free interest rate and a risk premium respectively. In this economy, equilibrium asset prices are given by the following Euler condition:

$$X_{t-1} = E^P\left[e^{-\rho}\frac{U'(C_t)}{U'(C_{t-1})}X_t|\Phi_{t-1}\right] \tag{2}$$

[1] We concur with Bates (1996), who prefers the term implicit volatility to implied volatility for grammatical reasons.

Under any of the three conditions spelled out in Duan's Theorem 2.1 (which amounts to $\frac{U'(C_t)}{U'(C_{t-1})} = e^{Y_t}$, where $Y_t = v + Z_t$, with v a constant mean and Z_t a $\aleph(0, \sigma_z^2)$ process under P), this condition can be rewritten as:

$$X_{t-1} = E^P \left[e^{-\rho + Y_t} X_t | \Phi_{t-1} \right] \tag{3}$$

In particular, the price of a European option which matures at time T is given by:

$$C_t = E^P \left[e^{-\rho(T-t) + \sum_{t=1}^{T} Y_t} Max(X_T - K, 0) | \Phi_t \right] \tag{4}$$

Now define a probability measure Q, absolutely continuous with respect to P, as:

$$dQ = e^{(r-\rho)T + \sum_{t=1}^{T} Y_t} dP \tag{5}$$

Then, for any Φ-measurable random variable W_t (see Lemma A.1 in Duan):

$$E^Q(W_t | \Phi_{t-1}) = E^P \left[W_t e^{(r-\rho) + Y_t} | \Phi_{t-1} \right] \tag{6}$$

Therefore, for $W_t = Max(X_T - K, 0)$, the option price is given under Q by:

$$C_t = e^{-r(T-t)} E^Q [\max(X_T - K, 0) | \Phi_t]. \tag{7}$$

Note that by the definition of Q and the Euler condition:

$$E^Q(X_t / X_{t-1} | \Phi_{t-1}) = e^r \tag{8}$$

Moreover, by the assumed conditional normality of Y_t:

$$Var^Q(\ln(X_t/X_{t-1}) | \Phi_{t-1}) = Var^P(\ln(X_t/X_{t-1}) | \Phi_{t-1}) \tag{9}$$

Conditions (8) and (9) define an extension of the concept of local risk neutral valuation intoduced by Rubinstein (1976) and Brennan (1979). Looking at a formula like (7), one might be tempted to consider it valid under global risk neutralization. However, as stressed by KT, "Viewed as discrete-time models, ARCH models do not allow for option pricing along the lines of Black and Scholes (1973), Cox, Ross, and Rubinstein (1979) and Harrison and Pliska (1981), because they are not complete". Taking into account the impossibility of completeness for markets in discrete-time settings where the unique primitive asset involves too much variability to be characterized by a binomial tree (as this is the case for a lognormal ARCH price process), KT suggest a suitable continuous time extension of ARCH-type processes.

The price process in continuous time X_t, $t \geq 0$, is then defined by:

$$X_t = X_0 \exp\left[\int_0^t (\mu(\sigma_u) - \sigma_u^2/2)du + \int_0^t \sigma_u dW_u\right] \tag{10}$$

where W_u is the standard Wiener process and the volatility process σ_u is a deterministic function of past prices, $X_\tau, \tau \leq u$. For instance, in the GARCH(1,1) spirit, KT propose:

$$\sigma_t = \begin{cases} \sigma_0 & for\ 0 \leq t < 1 \\ (\omega + \alpha(X_{[t]} - X_{[t]-1})^2 + \beta\sigma^2_{[t]-1})^{1/2} & for\ t \geq 1 \end{cases} \tag{11}$$

where $[t]$ denotes the greatest integer smaller than or equal to t. In other words, the basic idea of this continuous time extension of GARCH-type models is to maintain a constant volatility during an interval formed by two integer dates. The continuous time setting restores the completeness needed for arbitrage-based option pricing. In this setting, KT arrive at the same pricing formula (7) as Duan.

However, KT and Duan do not concur on the hedging formula. Perhaps due to the local risk-neutral valuation argument, Duan retains an unusual definition of delta hedging. The delta of an option is said to be "the first partial derivative of the option price with respect to the underlying asset price", and Corollary 2.4 states the following formula:

$$\Delta_t = e^{-r(T-t)} E^Q[\frac{X_T}{X_t}\mathbf{1}_{[X_T \geq K]})|\Phi_t] \tag{12}$$

Therefore, in the proof of Corollary 2.4 provided in the Appendix on p. 28, the author ignores the fact that C_t is also a function of the conditional variance of $\ln X_T$, itself a function of X_t in the GARCH(p,q) chosen by Duan. KT derive the usual delta of an option (the partial derivative of the option price with respect to the stock price), which depends explicitely on the volatility of the underlying asset and provides the optimal hedge. Despite its correctness, the hedging formula derived by KT appears counterintuitive in the sense that the hedging ratio depends on the last integer time volatility, however remote this time may be. Moreover, as we argue in section 3 below, the setting chosen by KT to bridge the ARCH discrete time setting to a continuous time model is not the best even in the family of ARCH models. The modern way to close the GARCH gap (see Drost and Werker (1996), and Meddahi and Renault (1996)) is based on the observation that when a continuous time stochastic volatility model is discretely sampled, a weak GARCH process is obtained. This way to cast discrete-time GARCH models in continuous time settings is much more appealing to capture stylized facts like the leverage effect. The stochastic volatility model also offers a way to reconcile a pricing result similar to KT with the hedging formula of Duan, which is more appealing since it is simpler to implement in practice.

2 Homogeneous Option Pricing and Hedging in Stochastic Volatility Models

We show below that, in order to obtain the simpler hedging formula (12), it is necessary and sufficient for the option pricing function to be homogeneous of degree one with respect to the pair (S_t, K). We argue further that this homogeneity property is desirable because it ensures that the option price is convex with respect to the underlying asset price, a property that is consistent with actual data (see Broadie et al. (1995)). Two additional useful implications of this homogeneity property are also pointed out.

Under the usual absence of arbitrage argument, there exists a pricing probability mesure Q under which option prices can be written as the discounted expected value of future payoffs:

$$C_t = e^{-r(T-t)} E_t^Q (S_T - K)^+ \tag{13}$$

A formula such as (13) provides a decomposition of the option price into two components:

$$C_t = S_t \Delta_{1t} + K \Delta_{2t} \tag{14}$$

where:

$$\Delta_{1t} = e^{-r(T-t)} E_t^Q \left[\frac{S_T}{S_t} \mathbf{1}_{\left[\frac{S_T}{S_t} \geq \frac{K}{S_t}\right]} \right] \tag{15}$$

$$\Delta_{2t} = -e^{-r(T-t)} Q_t \left[\frac{S_T}{S_t} \geq \frac{K}{S_t} \right] \tag{16}$$

It follows immediately (see Huang and Litzenberger (1988)) that:

$$\Delta_{2t} = \frac{\partial C_t}{\partial K} \tag{17}$$

This explains the equivalence between knowing option prices for any value of the strike price K (in other words the mapping K$\longrightarrow C_t$) and knowing the probability distribution (under Q) of the return $\frac{S_T}{S_t}$ (in other words the mapping $x \longrightarrow Q_t\left[\frac{S_T}{S_t} \geq x\right]$). Given this equivalence, useful properties of the option pricing formula (13) can be characterized by corresponding properties of the pricing probability measure Q_t. A property of special interest is the homogeneity of degree one of the option price with respect to the pair (S_t, K). By considering simultaneously (14) and (17) and the Euler characterization of the homogeneity property $(S_t \frac{\partial C_t}{\partial S}(S_t, K) + K \frac{\partial C_t}{\partial K}(S_t, K) = C_t)$, we can state the following proposition.

Proposition: The option pricing function is homogeneous of degree one with respect to the pair (S_t, K) if and only if the Δ hedging ratio of the option $\Delta_t = \frac{\partial C_t}{\partial S_t}$ is given by :

$$\Delta_t = \Delta_{1t} = e^{-r(T-t)} E_t^Q \left[\frac{S_T}{S_t} \mathbf{1}_{\left[\frac{S_T}{S_t} \geq \frac{K}{S_t}\right]} \right] \tag{18}$$

This is exactly the Δ hedging ratio formula proposed by Duan and reported in (12), but as explained above, KT showed that this is not the right hedging formula in the GARCH option pricing framework.

Indeed, the above homogeneity property is not inconsistent with the so-called volatility smile effect documented in Duan (1995). For instance, Garcia and Renault (1995) have set forth a stochastic volatility framework where both homogeneity and smile are captured. They prove that the key point to ensure homogeneity is an exogeneity property of both interest rate and stochastic volatility processes in the risk neutral dynamics. As an example, let us consider a risk neutral framework of a Markovian process (S, r, σ) defined by the following diffusion equations:

$$\begin{gathered} \frac{dS_t}{S_t} = r(t)dt + \sigma(t)dW^s(t) \\ dr(t) = \alpha(t)dt + \beta(t)dW^r(t) \\ d\sigma(t) = \gamma(t)dt + \delta(t)dW^\sigma(t) \end{gathered}$$

$$Var \begin{bmatrix} dW^s(t) \\ dW^r(t) \\ dW^\sigma(t) \end{bmatrix} = \begin{bmatrix} 1 & \rho_{sr}(t) & \rho_{s\sigma}(t) \\ \rho_{sr}(t) & 1 & \rho_{r\sigma}(t) \\ \rho_{s\sigma}(t) & \rho_{r\sigma}(t) & 1 \end{bmatrix} dt \tag{19}$$

where $\alpha(t), \beta(t), \gamma(t), \delta(t), \rho_{sr}(t), \rho_{s\sigma}(t), \rho_{r\sigma}(t)$ are $\mathrm{I}_t = \sigma[W^r(\tau), W^\sigma(\tau), W^S(\tau)]$ adapted stochastic processes. In this setting, a sufficient condition for the homogeneity of option prices (see Garcia and Renault (1995)) is that the mean, variance and covariance processes are deterministic functions of the processes r and σ. It is to be noted that leverage effects ($\rho_{s\sigma} \neq 0$) and cross-correlations between the stock price and the interest rate ($\rho_{sr} \neq 0$) are allowed provided that they do not depend on the level of the stock price. Due to these correlations, the state variables r and σ may not be independent from W^s but they are exogenous in the sense that their dynamics can be defined without any reference to S (in particular the process (r, σ) is Markovian).[2]

Beside ensuring homogeneity, this framework is able to produce a smile effect (see Hull and White (1987) and Renault and Touzi (1996)) since, even when skewed by the leverage effect, the option pricing formula is close to an expectation of the usual Black-Scholes (BS) formula, as it is the case in Duan. Indeed, it is worth noting that with T=t+2, the Duan's option pricing formula (7) can be reinterpreted as an expectation of the BS price computed with the conditional variance at time t+1[3].

In other words, the stochastic volatility paradigm is able to produce a smile effect as in the endogenous volatility paradigm developed recently in the option pricing literature by Rubinstein (1994) and Duffie (1995) among others (implied tree models). These endogeneous volatility models, where the volatility process $\sigma(t)$ is viewed as a deterministic function of the past history of the underlying asset price, $\mathrm{S}_\tau, \tau \leq t$ as in GARCH-type models (as opposed to stochastic volatility models), have the disadvantage of losing the homogeneity property of option prices.

Apart from leading to a simpler hedging formula, the homogeneity property is, as noticed by Merton (1973)[4], a natural way to ensure the convexity of the option price with respect to the underlying asset price[5]. Using data on the S&P 100 option prices, Broadie et al. (1995) have estimated non-parametrically the function linking the option price to

[2]This characterization of the homogeneity property generalizes Merton (1973), who showed that serial independence of asset returns is a sufficient condition for homogeneity.

[3]Iterating further will not provide such a simple interpretation since it is well-known that GARCH dynamics do not allow to compute multi-step forecasts of variances.

[4]Merton (1973) stresses that "... convexity is usually assumed to be a property which always holds for warrants", but he is able to provide an example where the distribution of future returns is "sufficiently dependent on the level of the stock price to cause perverse local concavity".

[5]Bergman, Grundy and Wiener (1996) propose a sufficient condition for convexity of option prices in a stochastic volatility model which corresponds exactly to the Garcia and Renault characterization of homogeneity referred to above.

the stock price and it is clearly convex. Moreover, this convexity means that the delta ratio is an increasing function of the underlying asset price, which is consistent with the empirically established destabilizing effect of portfolio insurance strategies during the last stock market crash of 1987[6].

Two other implications of the homogeneity property are also useful to understand the way practitionners use the BS implicit volatility $\sigma_t(S_t, K)$ defined for given maturity and interest rate by:

$$C_t = BS(S_t, K, \sigma_t(S_t, K))$$

where $BS(S_t, K, \sigma)$ denotes the BS price associated to a volatility parameter σ. First, by the Euler characterization of the homogeneity property, the delta ratio can be written as:

$$\Delta_t = \frac{\partial C_t}{\partial S_t} = \frac{\partial BS}{\partial S}(.) - \frac{\partial BS}{\partial \sigma}(.)\frac{K}{S_t}\frac{\partial \sigma_t}{\partial K}(.) \tag{20}$$

Since $\frac{\partial BS}{\partial \sigma}(.) > 0$, equation (20) shows that the two expressions $\Delta_t - \frac{\partial BS}{\partial S}(S_t, K, \sigma_t(S_t, K))$ and $\frac{\partial \sigma_t}{\partial K}(.)$ are of opposite signs. This provides a useful relationship since the first expression measures the hedging error due to the misspecification of the BS option pricing model used to infer the implicit volatility and the second expression is the "smile effect" which characterizes the variations of the BS implicit volatility σ_t as a function of the strike price K. In other words the smile effect measures the hedging error made when hedging is based on the Black-Scholes formula.

The second implication is a rationalization of option quotations since practitioners are interested primarily by the percentage $x_t = \log \frac{S_t}{K}$ of in-the-moneyness or out-of-the-moneyness of the options. Indeed, the homogeneity property ensures that the Black-Scholes (BS) implicit volatility, $\sigma_t(S_t, K)$, depends on (S_t, K) only through x_t.

We are then faced with two alternative classes of option pricing models. In the first, by chosing the most traditional setting for conditional heteroskedasticity, i.e. the GARCH-type model, we lose the homogeneity property. With the second, the class of stochastic volatility (SV) models which were mainly introduced in the literature for option pricing purposes (see Hull and White (1987), we can retain the appealing homogeneity property. Until recently, these two classes of models were generally perceived as competitors, especially because of the difficulty

[6] When the delta ratio is an increasing function of the underlying stock price, people who perform portfolio insurance buy (resp. sell) the stock when its price increases (resp. decreases).

of estimating SV models. Recent advances in econometric theory have made estimation of SV models much easier (see Ghysels, Harvey and Renault (1996) for a survey) and have redefined the terms of competition between these two types of models.

3 Stochastic Volatility versus ARCH Option Pricing Models

The loss of the homogeneity property in usual discrete-time statistical models like ARCH-type models is not as damaging as it appears for several reasons. First, Nelson (1990) has shown that the distinction between "stochastic volatility" (where the source of randomness in the underlying asset volatility is exogenous) and endogenous volatility is not robust to temporal aggregation. ARCH-type discrete-time models may converge towards stochastic volatility models in continuous time as the time interval goes to zero. Second, a number of studies (see Drost and Werker (1996), Meddahi and Renault (1996) and Ghysels, Harvey and Renault (1996)) have shown that the class of GARCH processes which is robust to temporal aggregation, namely the weak GARCH class (see Drost and Nijman (1993)), is a sub-class of stochastic volatility models. In particular, when we sample in discrete time a continuous SV model, we obtain a weak GARCH model. Therefore, ARCH-type models and SV models are not competitors (as it was commonly believed) but rather complements, since the ARCH model offers a useful discrete-time filter for SV models.

Of course, it is always possible, as shown by KT, to cast ARCH-type models in a continuous time setting for option pricing, given a certain discrete-time sampling scheme, without studying the limit as the time interval goes to zero. However, it appears as a rather artificial construct and results in non-robust hedging ratios as mentioned above. More generally, as explained by Rubinstein (1994), implied tree models are usually good for pricing options but not for hedging. Among the four categories of violations to the constant volatility Black-Scholes model that he refers to, only the less serious one, whereby the local volatility of the underlying asset is a function of the concurrent underlying asset price, allows for a computationally effective way to price as well as to hedge options. More serious violations, like the dependence on past asset prices and a fortiori on exogenous state variables, invalidate the methodology for hedging purposes. In this regard, in the stochastic volatility framework, we are interested in two hedging ratios, $\frac{\Delta C_t}{\Delta S_t}$ and

$\frac{\Delta C_t}{\Delta \sigma_t}$, associated with the two sources of risk. The use of an ARCH-type filter in discrete time leads to a spurious integration of the volatility risk in the Δ ratio, in the sense that the resulting Δ ratio wrongly mixes the correct Δ in (12) and $\frac{\Delta C_t}{\Delta \sigma_t}$.

4 Concluding Comments

To summarize, the endogeneous or exogenous characterization of the source of randomness introduced in the asset price volatility to explain the smile is not as clear-cut as it appears. Apart from the fact that both SV models and GARCH-type models can reproduce the stylized facts associated with the smile according to the simulations performed by Hull and White (1987) and Duan (1995), we argued that these models are not as far apart as originally believed. The clear distinction appears in the use of these models for hedging purposes, since the formula derived under GARCH-type models loses the homogeneity property referred to above. We argue that this distinctive feature makes stochastic volatility models more attractive.

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